Structure Functions in $\tau \to 3\pi\nu$

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Introduction

For the τ semileptonic decay

$$\tau(l,s) \rightarrow \nu(l',s') + \pi_1(q_1) + \pi_2(q_2) + \pi_3(q_3) \tag{1.1}$$

The matrix element is written as

$$\mathcal{M} = cos\theta_c \frac{G_F}{\sqrt{2}} M_\mu J^\mu \tag{1.2}$$

The differential decay rate is

$$d\Gamma(\tau \rightarrow 3\pi\nu) = \frac{G_F^2 cos^2\theta_c}{4m_\tau}[L_{\mu\nu}H^{\mu\nu}]dPS^{(4)} \tag{1.3}$$

the lepton and hadron tensors are hermitian and can be expanded in a sum of 16 independent terms,

$$L_{\mu\nu}H^{\mu\nu} = \sum_X L_X W_X = (g_V^2 + g_A^2)(m_\tau^2 - Q^2) \sum_X \overline{L}_X W_X \tag{1.4}$$

where

$$X \in [A, B, C, D, E, F, G, H, I, SA, SB, SC, SD, SE, SF, SG]$$

$W_X(s_1, s_2, Q^2)$ is so called hadronic structure functions.
$\overline{L}_X(\alpha, \gamma, \beta, \theta, Q^2)$ relatives to leptonic current, it is determined by Standard Model.
so eq.(1.3) can be rewritten as

$$\frac{d\Gamma(\tau \rightarrow 3\pi\nu)}{dQ^2} = f(Q^2) \sum_X [(\overline{L}_X d\gamma dcos\beta dcos\theta)(W_X ds_1 ds_2)] \tag{1.5}$$

Theoretical Models

- **Kuhn - Santamaria (KS) Model**

$$J_\mu = J_\mu^{(1)} + J_\mu^{(0)} \tag{2.1}$$

Characteristics:

1) it bases on assumption of chiral dynamics

2) τ decay to a_1 (spin 1) or π' (spin 0)

3) contribution from $\pi' < 5\%$

Assuming G-parity invariance $\Longrightarrow W_{B,F,G,H,I,SF} = 0$

Integrating on α, $L_{SC} = L_{SE} = L_{SG} = 0$.

7 structure functions exist, they are

$$\begin{pmatrix} W_A, W_C, W_D, W_E - spin\ 1 \\ W_{SA}, W_{SB}, W_{SD} - spin\ 0 \end{pmatrix}$$

Theoretical Models

- **Isgur - Morningstar - Reader (IMR) Model**

$$J_\mu = J_\mu^{(1)} + J_\mu^{(0)} \tag{2.2}$$

Characteristics:

1) it relies on flux-tube-breaking model

2) τ decays to a_1 (spin 1) or π' (spin 0)

3) it introduces strong form-factors

4) running M_{a_1} in Breit Wigner formula

5) $a_1 \to \rho\pi$ with D-wave and S-wave, prediction is

$$f^D/f^S = -0.12, (ARGUS : -0.11 \pm 0.02)$$

6) it has polynomial background term - in order to consider possible small effects (OPAL: $c_1 = -7.3, c_2 = 14.9, c_3 = -1.9$)

$$\frac{d\Gamma_b}{dQ^2} = \frac{(m_\tau^2 - Q^2)^2}{Q^2} \sum_{n=1}^{3} c_n (Q^2 - 9m_\pi^2)^n$$

Also have 7 structure functions

$$\begin{pmatrix} W_A, W_C, W_D, W_E - spin\ 1 \\ W_{SA}, W_{SB}, W_{SD} - spin\ 0 \end{pmatrix}$$

Theoretical Models

- **M.Feindt (MF) Model**

$$J_\mu = aJ_\mu^{(S)} + bJ_\mu^{(D)} \tag{2.3}$$

Characteristics:

1) it uses angular momentum eigenstates amplitude

2) it has both D-wave and S-wave

3) it's only a general frame

4) it needs to combine with other models(eg., KS, IMR)

- **Li Bing-An (Li) Model**

$$J_\mu = J_\mu^{(1)} \tag{2.4}$$

Characteristics:

1) it bases on chiral theory of meson

2) without spin 0 component

3) $M_{a_1},\ \Gamma_{a_1}$ is given by theory itself

4) it has special Breit Wigner formula

Only have 4 structure functions

$$\Big(W_A, W_C, W_D, W_E - spin\ 1 \Big)$$

Comparisons of theoretical Models

1) w_A as a function of Q^2 (Fig.1)

2) $w_C/w_A, w_D/w_A, w_E/w_A$ as functions of Q^2 (Fig.2)

3) $w_{SA}/w_A, w_{SB}/w_A, w_{SD}/w_A$ as functions of Q^2 (Fig.3)

Models	M_{a_1}(GeV)	Γ_{a_1}(GeV)
KS	1.251	0.599
IMR	1.220	0.477
Li	1.201	0.386

Tab. 1: Important parameters of models.

Comparisons of theoretical Models

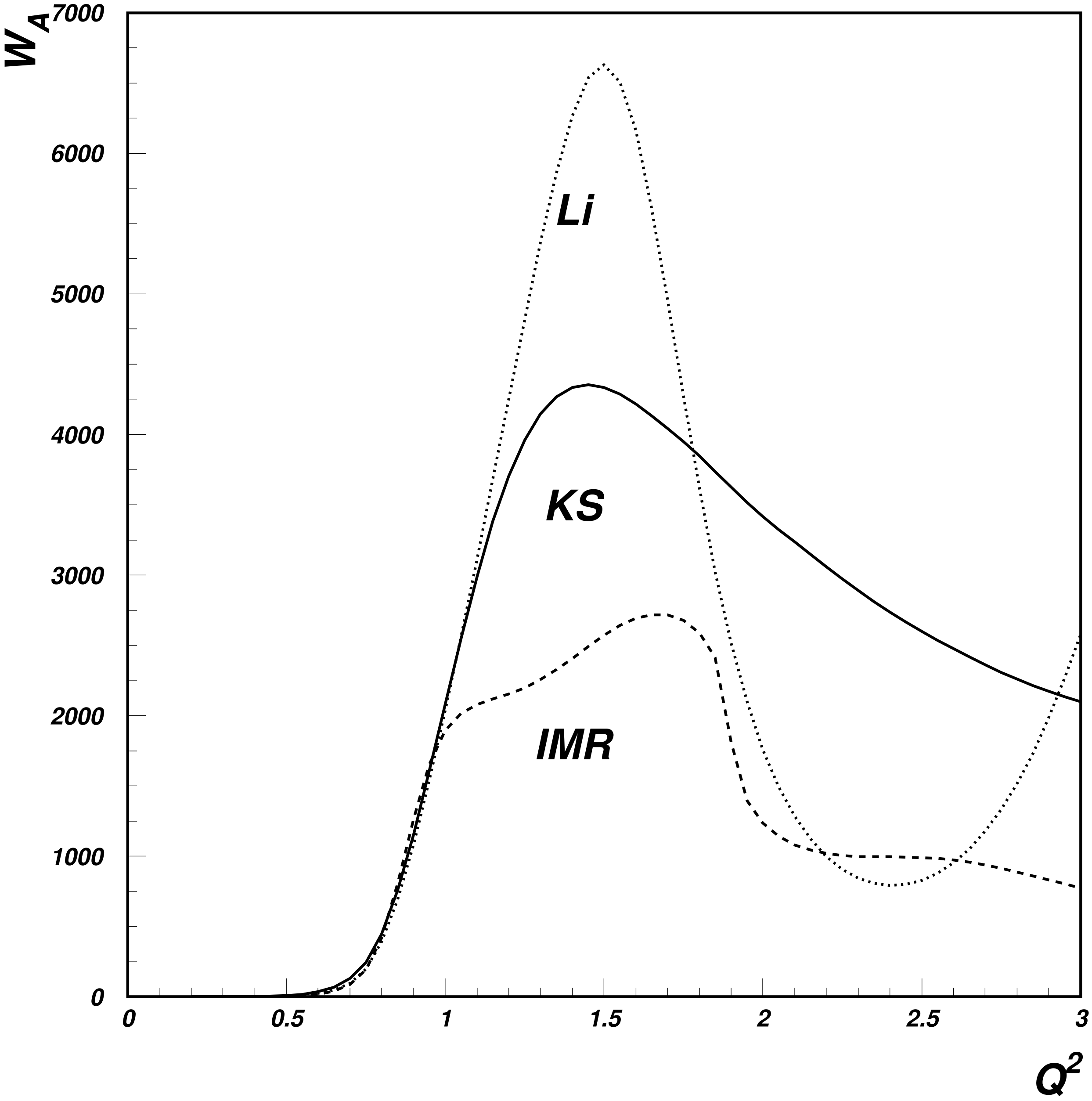


Fig. 1: w_A **as function of** Q^2**.**

Comparisons of theoretical Models

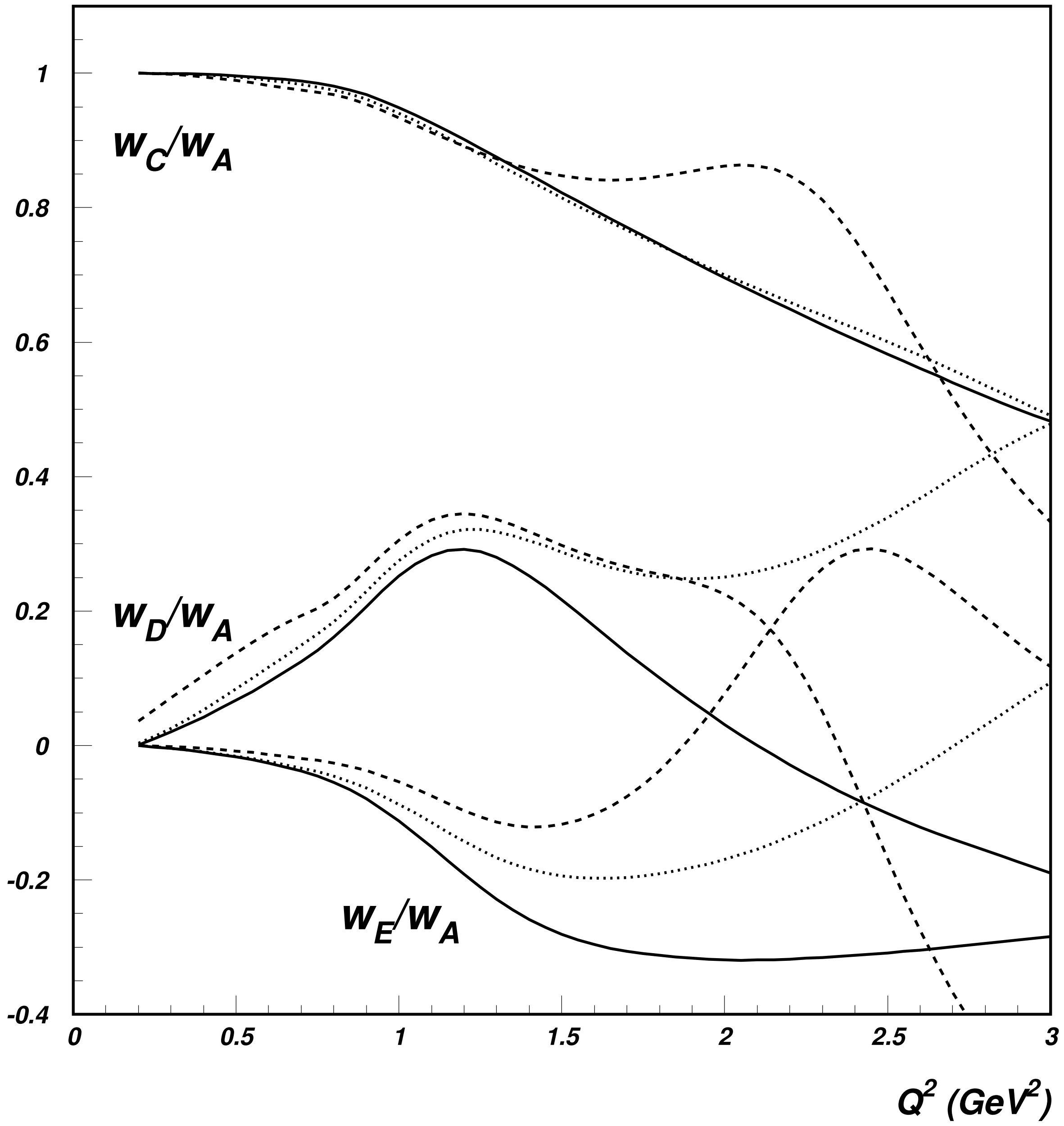


Fig. 2: $w_C/w_A,\ w_D/w_A,\ w_E/w_A$ **as function of** Q^2**. Solid lines represent KS Model predictions, dotted lines are IMR results, dashed lines are Li's predictions.**

Comparisons of theoretical Models

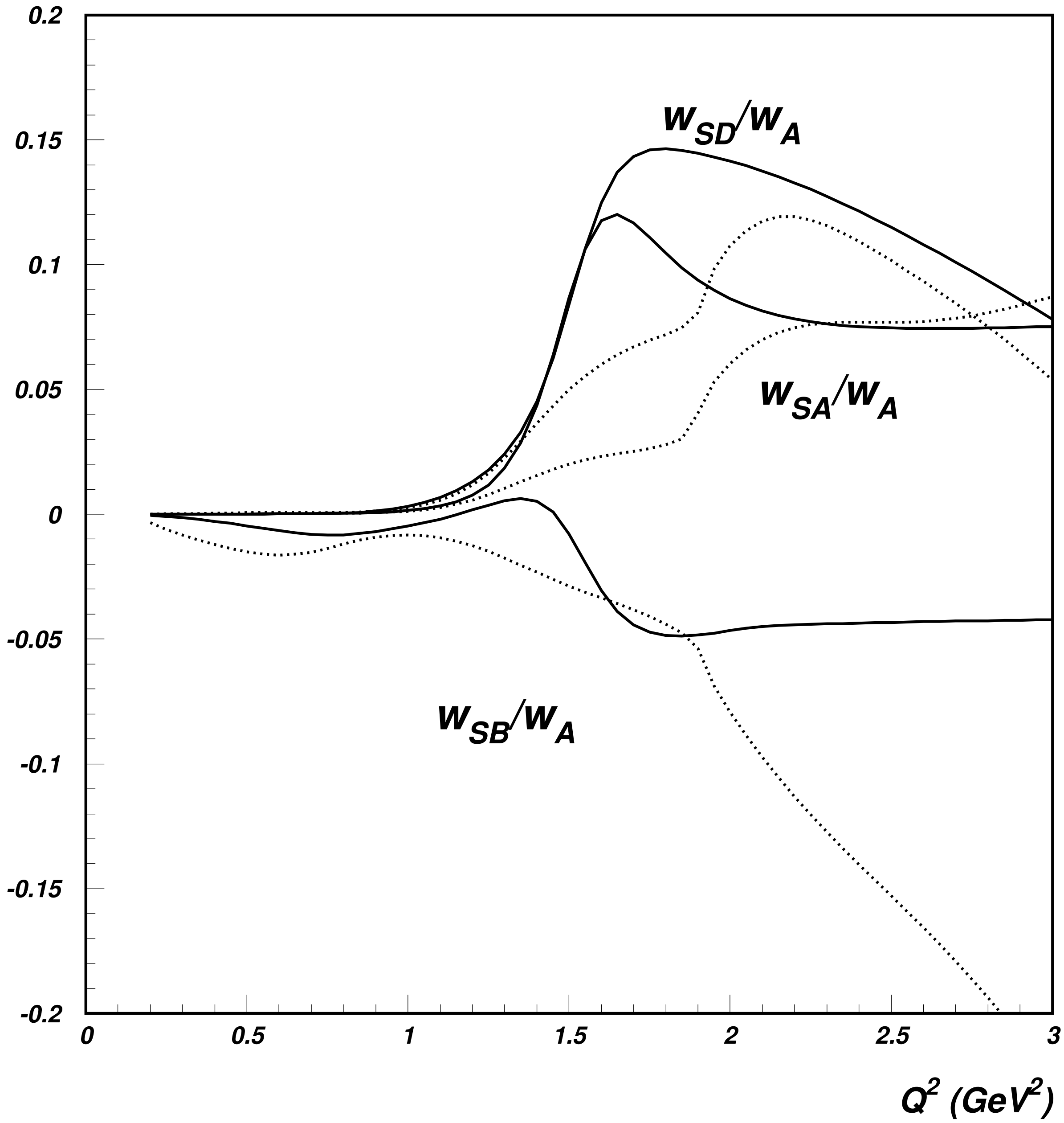


Fig. 3: $w_{SA}/w_A,\ w_{SB}/w_A,\ w_{SD}/w_A$ **as function of** Q^2**. Solid lines represent KS Model predictions, dotted lines are IMR results.**

Experimental Measurement

- **Assumption**

  Considering conservation of G-parity, $W_{B,F,G,H,I} = 0$ In PCAC, the contribution from scalar current (spin 0) is too small to be neglect. Hence, the number of structure functions is reduced to 4, they are $(W_A,\ W_C,\ W_D,\ W_E)$

- **Extraction of Structure functions**

  According to eq. (1.5), we obtain

$$\frac{d\Gamma}{dx}(x; w_X) = f(Q^2)(\overline{L}_A w_A + \overline{L}_C w_C + \overline{L}_D w_D + \overline{L}_E w_E) \tag{3.1}$$

$$where\ dx = d\gamma dcos\beta dcos\theta dQ^2,\ w_X = \int W_X ds_1 ds_2$$

$$\frac{N_{tot}}{\Gamma_{3\pi\nu}}\frac{d\Gamma}{dx}\ is\ expected\ events\ in\ x\ \sim x\ +\ dx$$

  After integration over $(\gamma, cos\beta, cos\theta)$ in full phase space,

$$\int \overline{L}_C d\gamma dcos\beta dcos\theta = 0$$

$$\int \overline{L}_D d\gamma dcos\beta dcos\theta = 0$$

$$\int \overline{L}_E d\gamma dcos\beta dcos\theta = 0$$

  So only 1 structure function w_A exists.(Fig.5).

Experimental Measurement

In order to fit other functions, we divide phase space into many bins, e.g.,
$3(\gamma) \times 3(cos\beta) \times 3(cos\theta) \times 10(Q^2) = 270$ bins.

- **Binned Maximum Likelihood method**

$$P(N^i_{obs}, N^i_{exp}(w_X)) = \frac{(N^i_{exp})^{N^i_{obs}}}{N^i_{obs}!} e^{-N^i_{exp}}$$

$$\mathcal{L} = \prod_{i=1}^{N_{bin}} P(N^i_{obs}, N^i_{exp}(w_X))$$

$$Max(\ln \mathcal{L}) \Longrightarrow w_A, w_C, w_D, w_E$$

where
N^i_{exp} is number of expected events in i bin
N^i_{obs} is number of real events in i bin

- **Extended Maximum Likelihood method**

$$\ln \mathcal{L} = \sum_{i=1}^{N_j} \ln\left(\frac{N_{tot}}{Corr\Gamma_{3\pi\nu}} \frac{d\Gamma}{dx}(x_i; w^j_X)\right) - \frac{N_{tot}}{Corr\Gamma_{3\pi\nu}} \Gamma(\Delta Q^2_j; w^j_A)$$

$$\Gamma(\Delta Q^2_j; w^j_A) = \int_{(\gamma, cos(\theta), cos(\beta), \Delta Q^2_j)} \frac{d\Gamma}{dx}(x_i; w^j_A) dx$$

Experimental Measurement

- **Potential Problem**

  If we want to compare data with theory predictions, we must take detector effects, efficiencies and acceptance into account. **Observed events**(data) should correct into **real events** in each bin. The correction factors can be estimated by using Monte Carlo events.

Correction Method

Here we use **a multidimensional unfolding method based on Bayes' theorem** (Ref: NIM A362(1995) 487-498) to unfold experimental distributions in order to get the **true** ones.

Let's state Bayes' theorem in terms of some independent **causes** ($C_i,\ i = 1, 2, ..., n_C$) which can produce some **effects** ($E_j,\ j = 1, 2, ..., n_E$). If we know the **initial probability** of the causes $P_0(C_i)$ and the **conditional probability** of the ith cause to produce the jth effect $P(E_j|C_i)$.

The Bayes formula is then

$$P(C_i|E_j) = \frac{P(E_j|C_i)P_0(C_i)}{\Sigma_{l=1}^{n_C} P(E_j|C_l)P0(C_l)} \qquad (1)$$

This means that if we observe a single event in jth effect, the probability that it has been due to the ith cause, which depend on the probability of the cause and the probability of the cause to produce the effect.

After N_{obs} experimental observations one obtains a distribution of frequencies $n(E) \equiv \{n(E_1),\ n(E_2), ...,\ n(E_{n_E})\}$. The expected number of events to be assigned to each of the causes

Correction Method

can be written as

$$\hat{n}(C_i) = \frac{1}{\epsilon_i} \sum_{j=1}^{n_E} n(E_j) P(C_i|E_j) \tag{2}$$

Where ϵ_i is the efficiency of detecting the cause C_i in any of the possible effects.

So the **true** total number of events is

$$\hat{N}_{true} = \sum_{i=1}^{n_C} \hat{n}(C_i)$$

The final probabilities of the causes is

$$\hat{P}(C_i) = \frac{\hat{n}(C_i)}{\hat{N}_{true}}$$

The unfolding can be performed through the following steps:

1) choose $P_0(C)$, hence $n_0(C_i) = P_0(C_i)N_{obs}$;

2) calculate $\hat{n}(C)$ and $\hat{P}(C)$;

3) make a χ^2 comparison between $\hat{n}(C)$ and $n_0(C)$;

4) replace $P_0(C)$ by $\hat{P}(C)$, $n_0(C)$ by $\hat{n}(C)$,goto step 2) to start again until the χ^2 is "small enough".

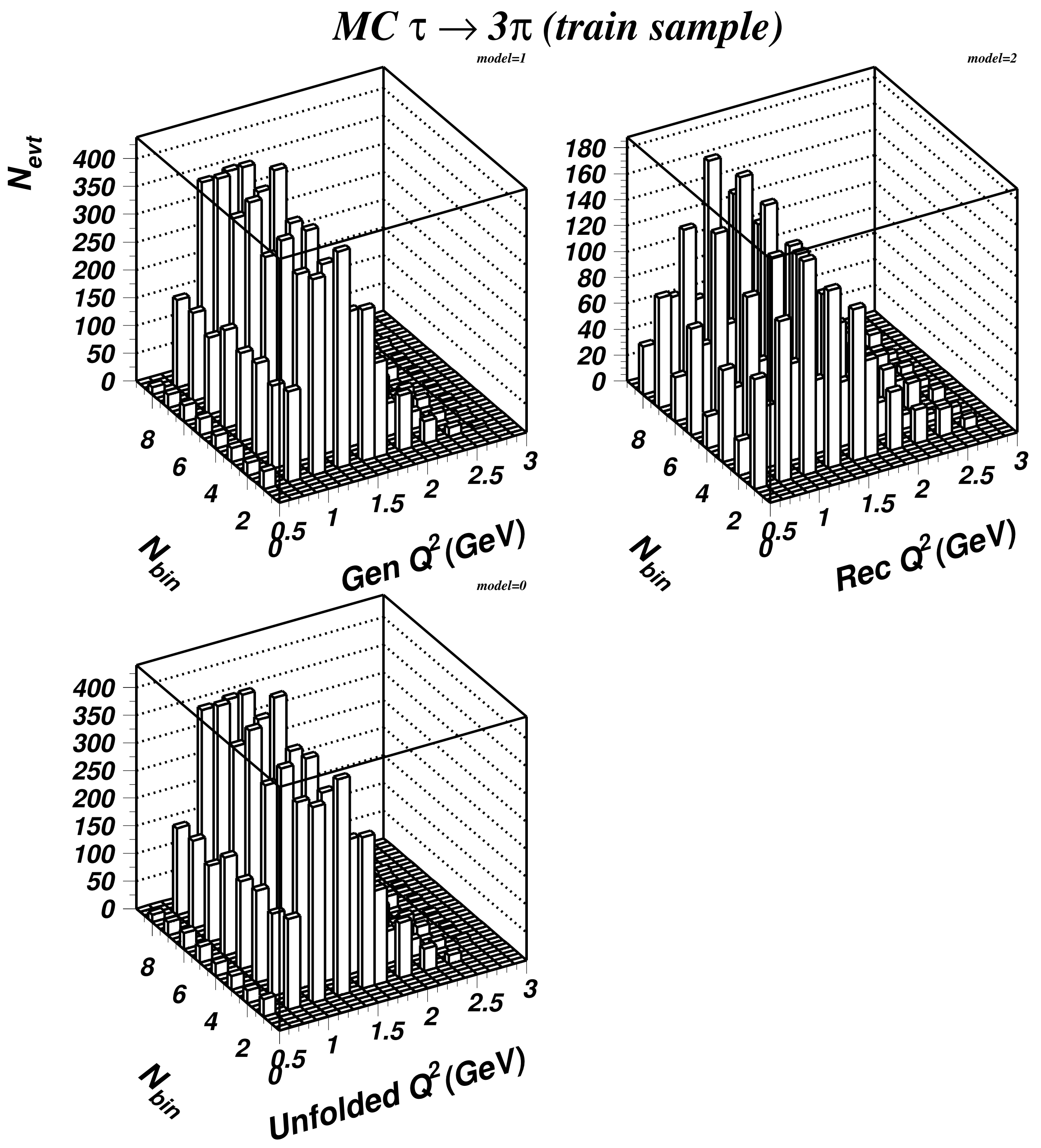


Fig. 4: **two dimensional distributions in Generator level(Gen), Reconstructed level(Rec) and unfolded ones.**

Correction Method

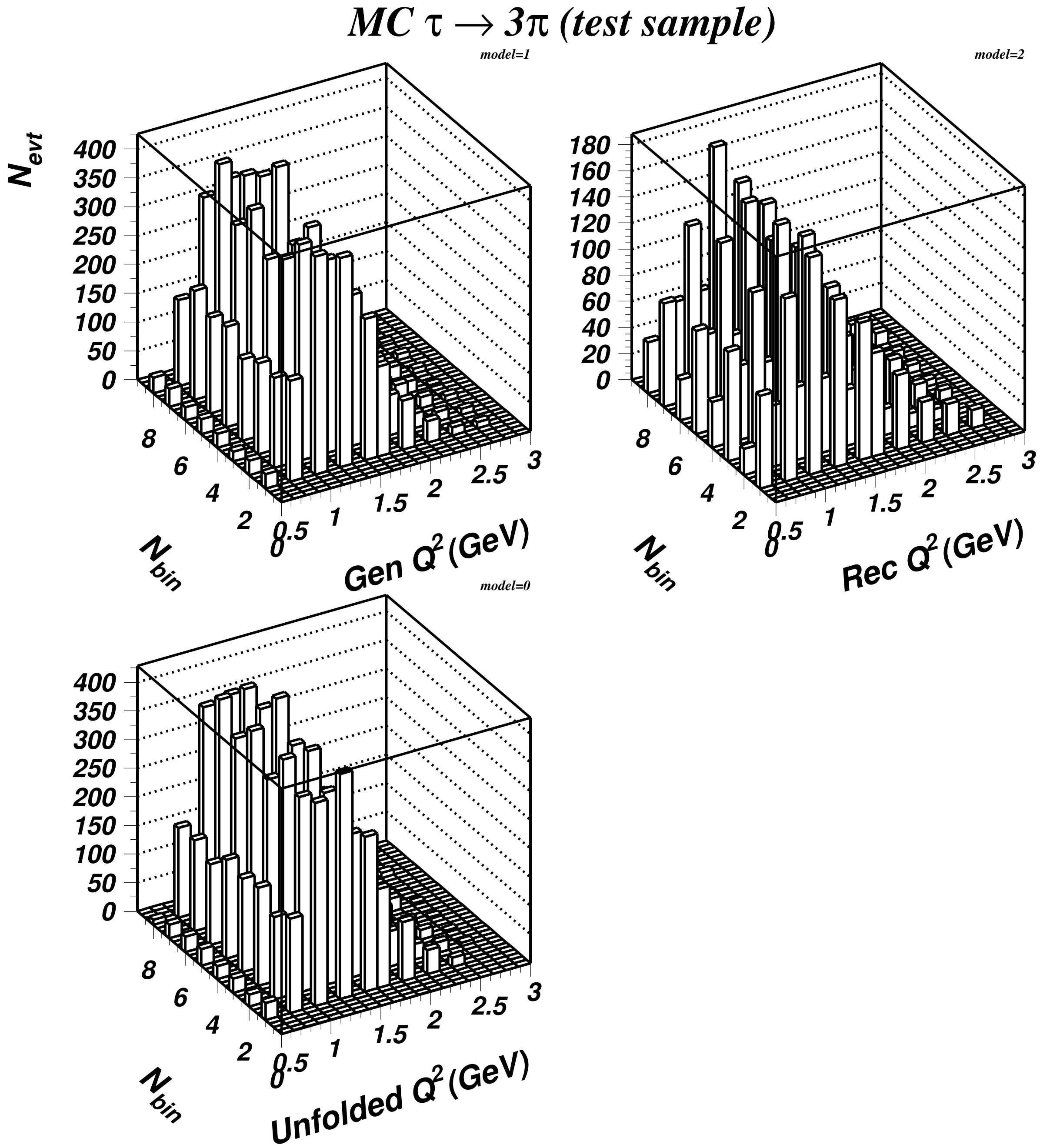
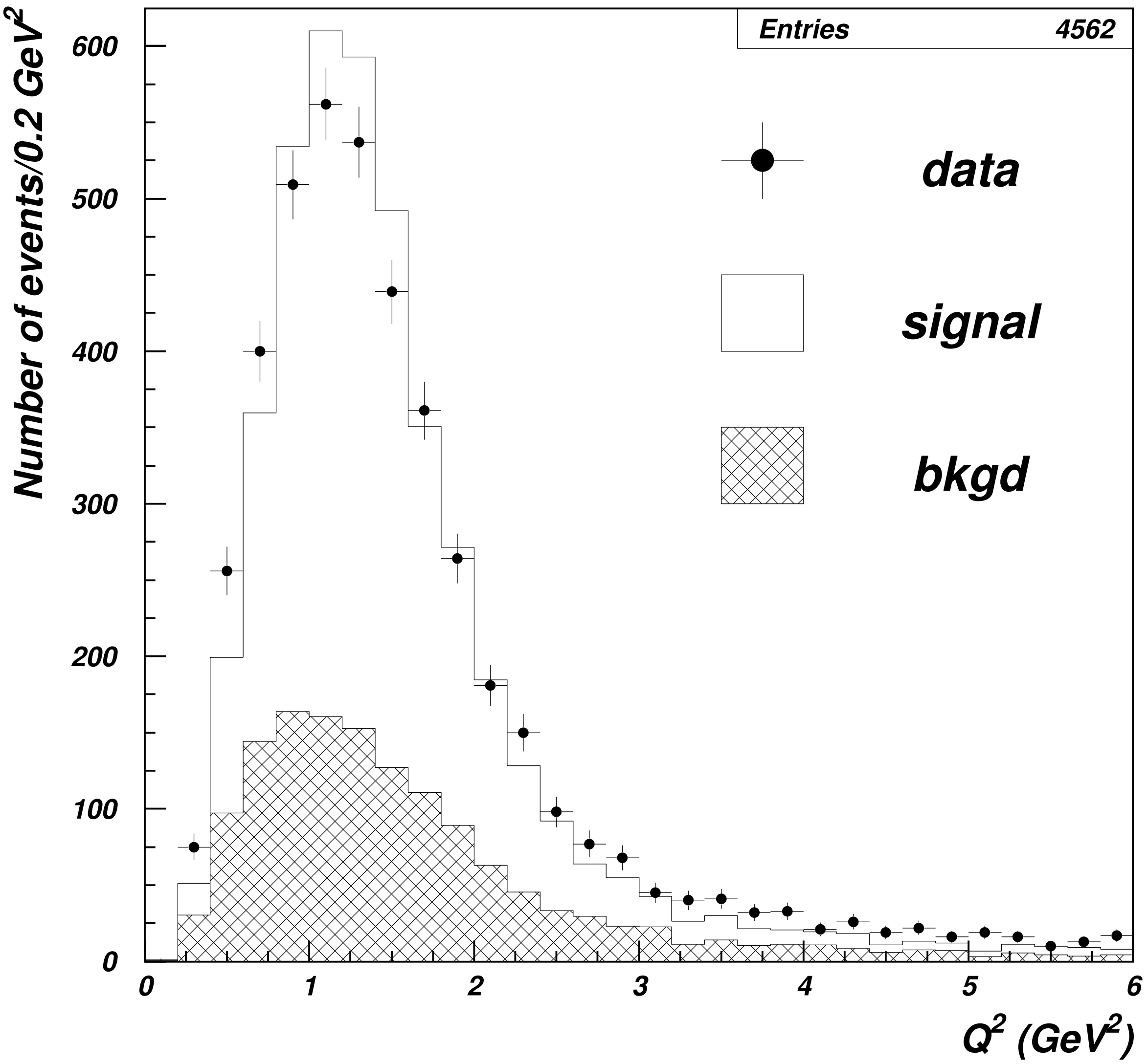


Fig. 5: **two dimensional distributions in Generator level(Gen), Reconstructed level(Rec) and unfolded ones.**

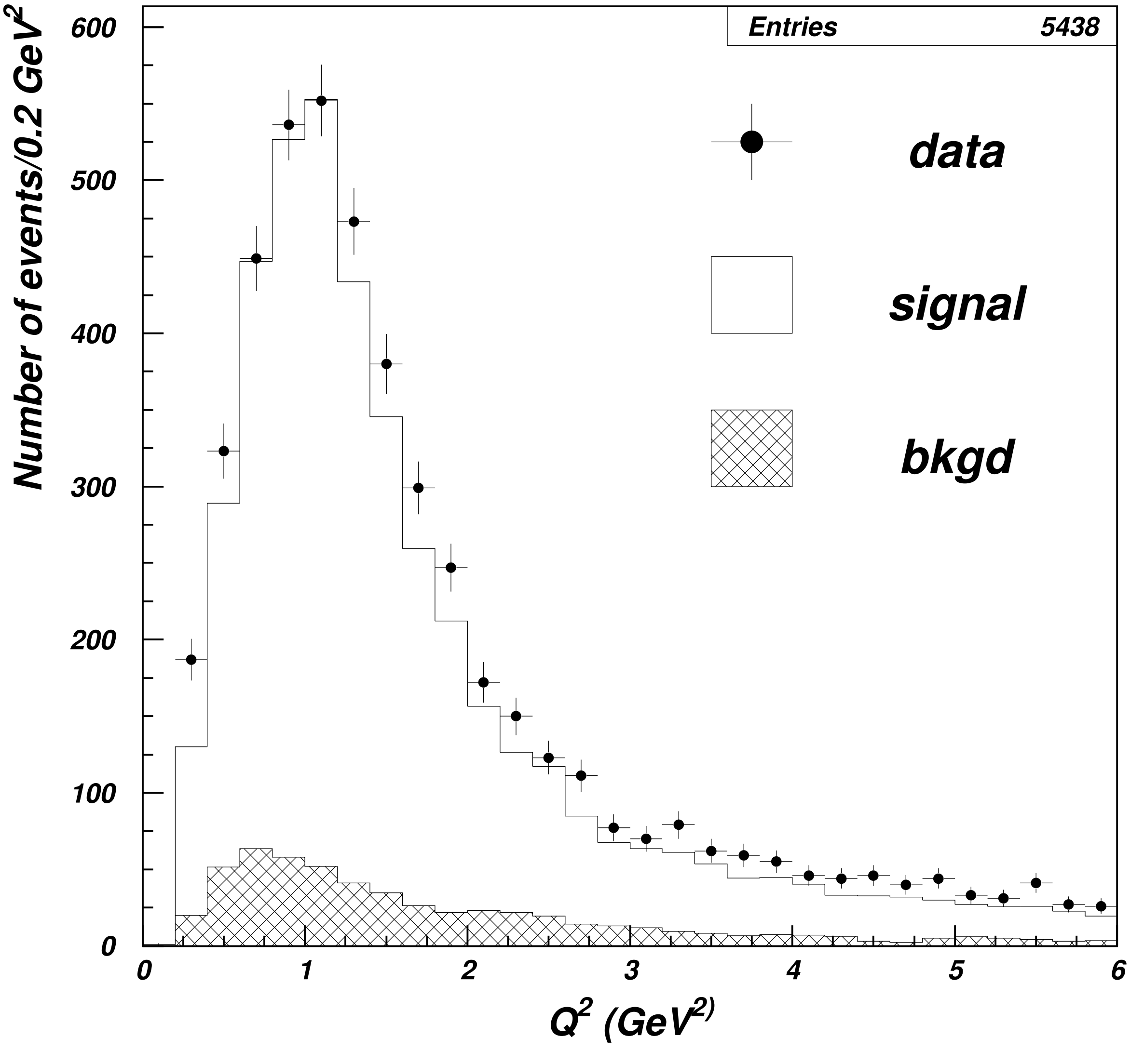
Data 92-95

$\tau^{\pm} \rightarrow \pi^{\pm}\, 2\pi^{0}\, \nu$ *(94)*

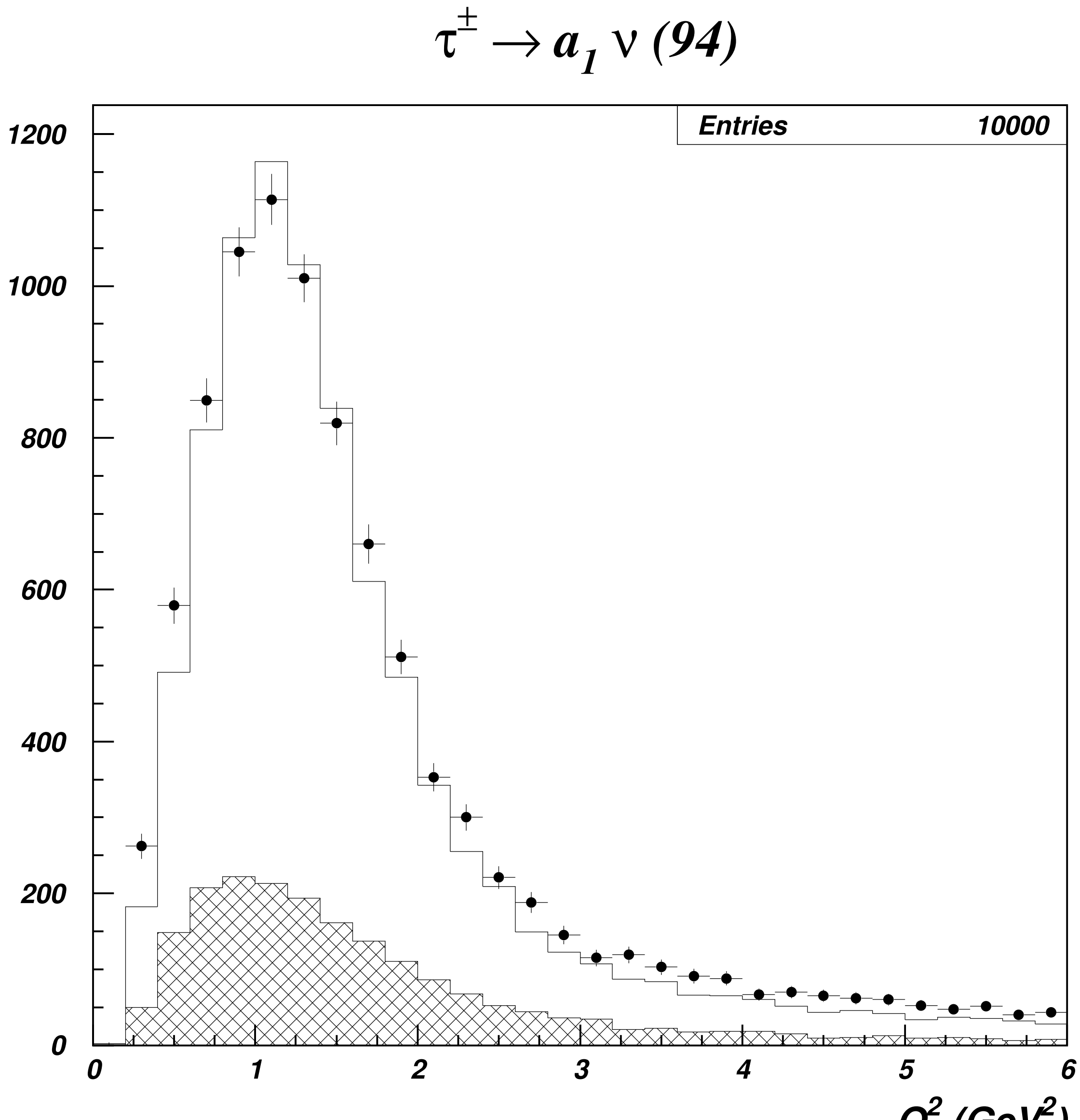


Data 92-95

$\tau^{\pm} \to 3\pi^{\pm}\nu$ *(94)*

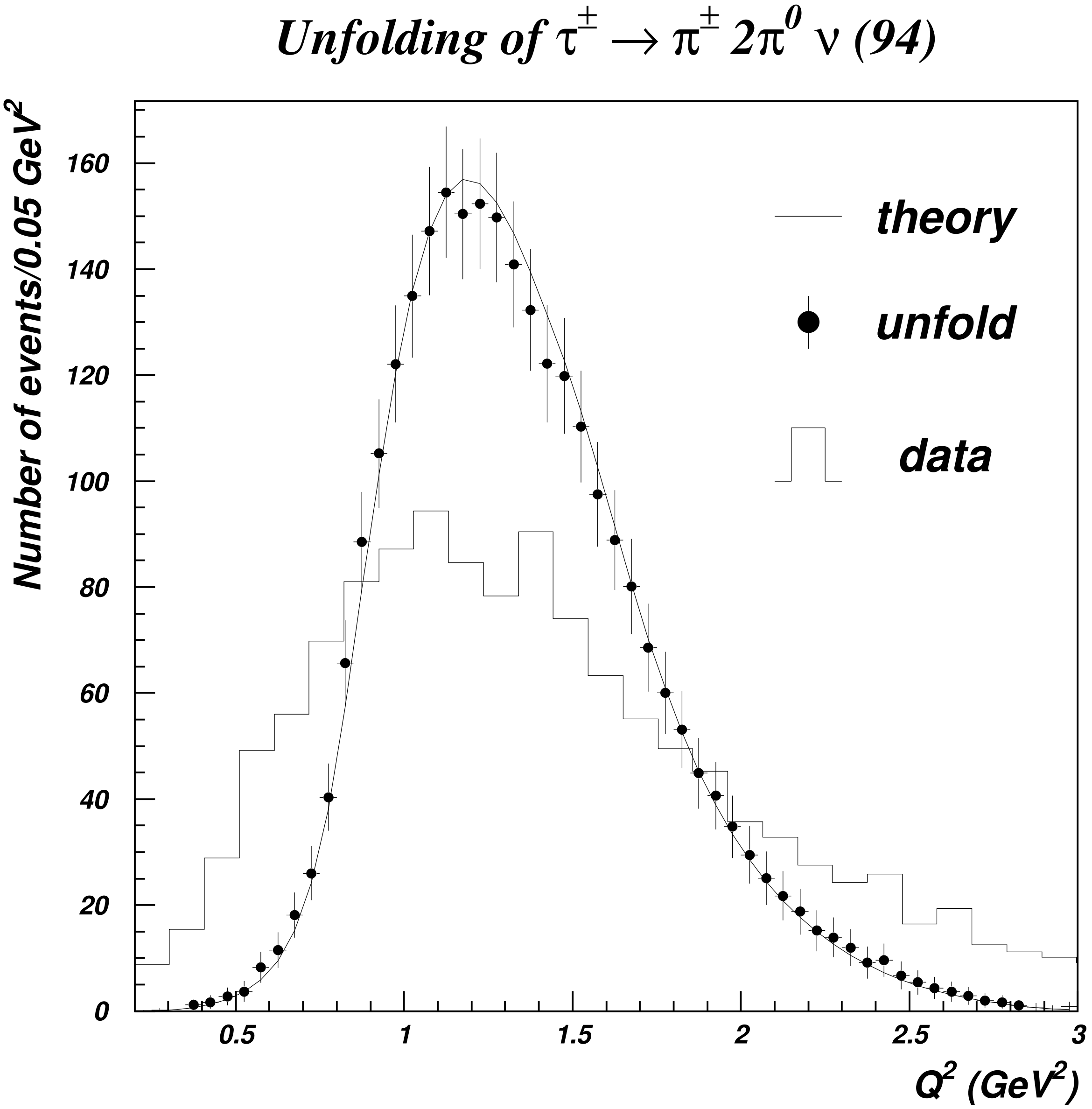


Data 92-95

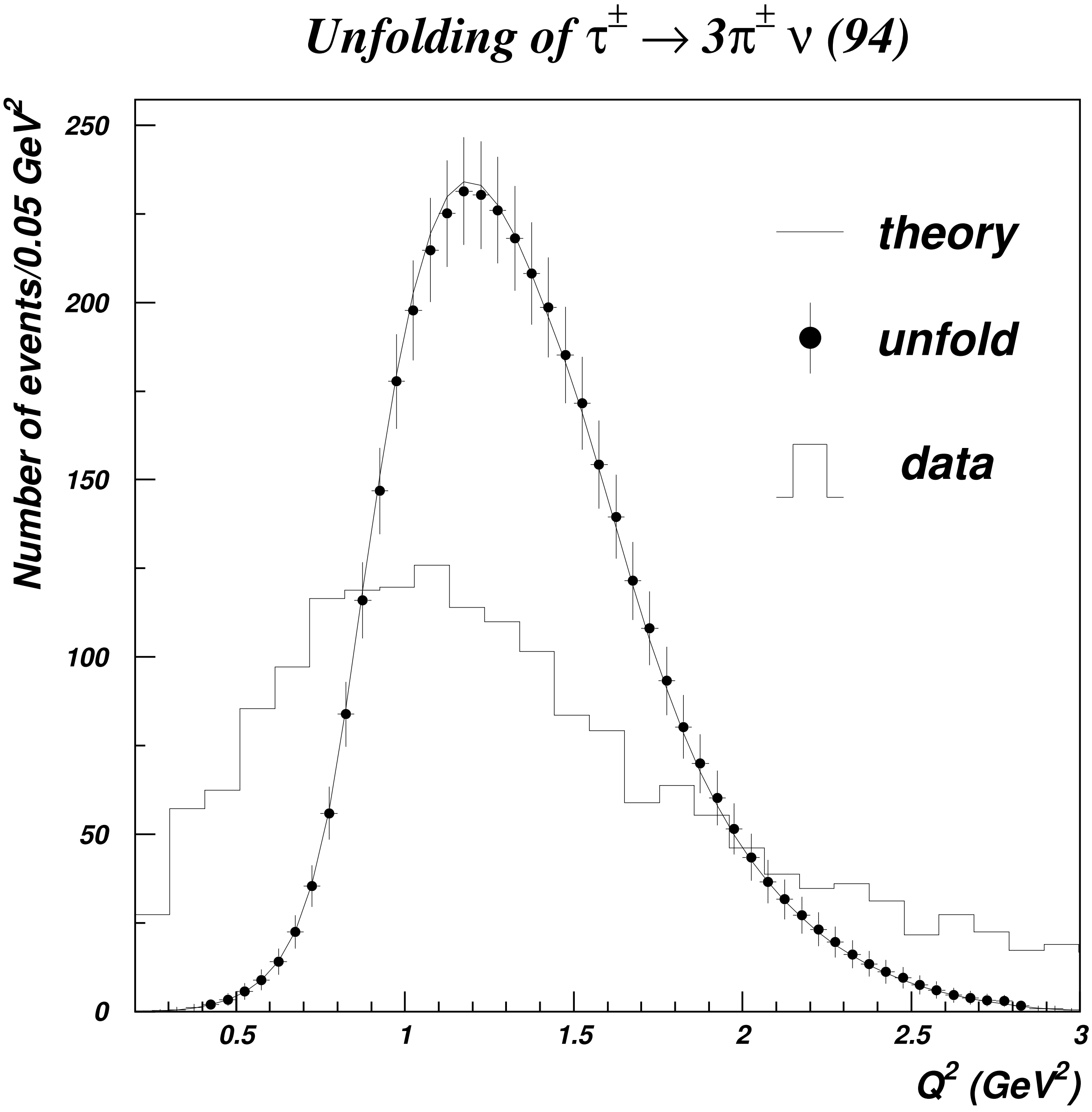


$\tau^{\pm} \to a_1\, \nu$ (94)
Entries
10000
0
200
400
600
800
1000
1200
0
1
2
3
4
5
6
Q^2 (GeV^2)

Unfolding Results



Unfolding Results



Unfolding Results

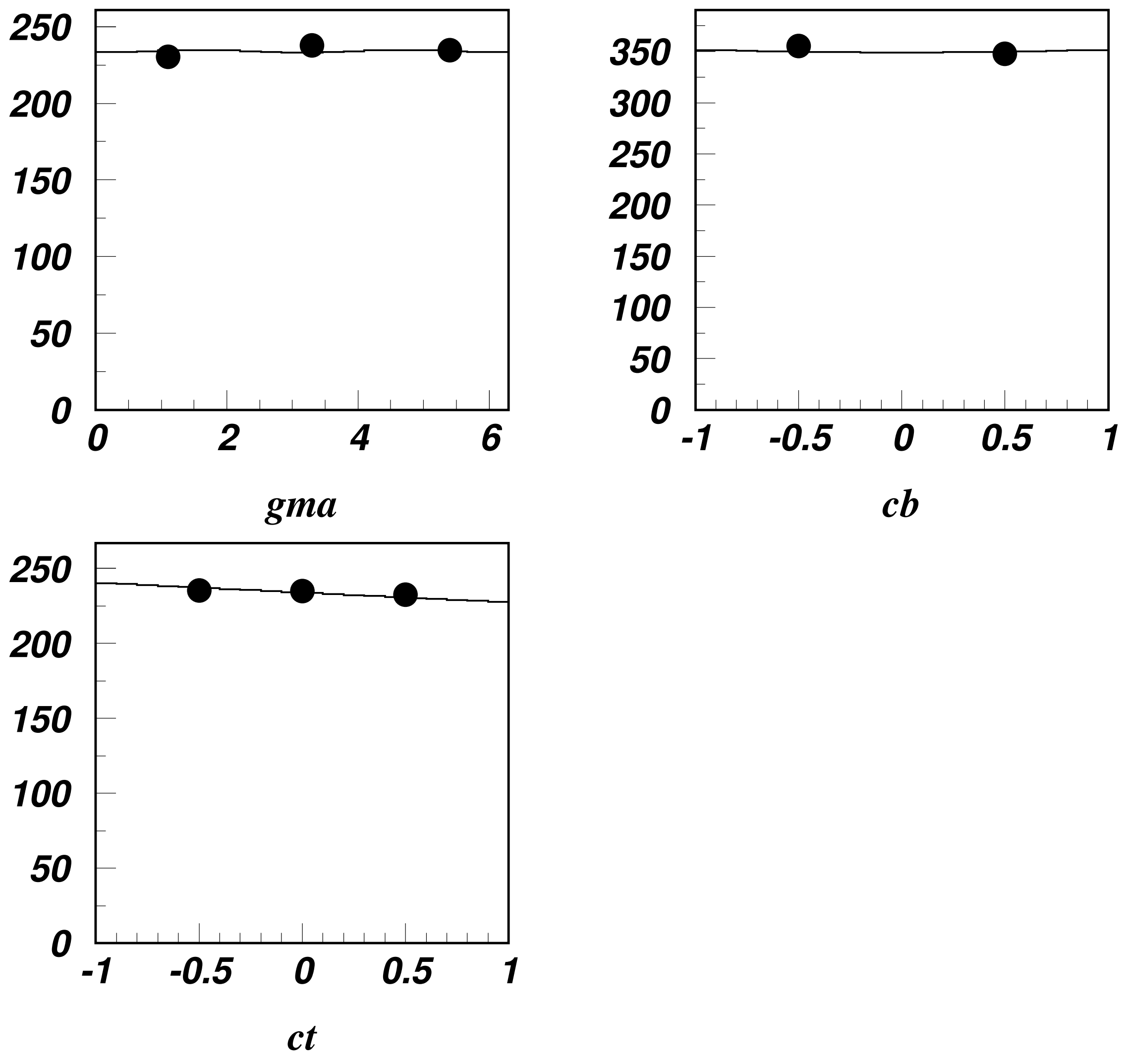
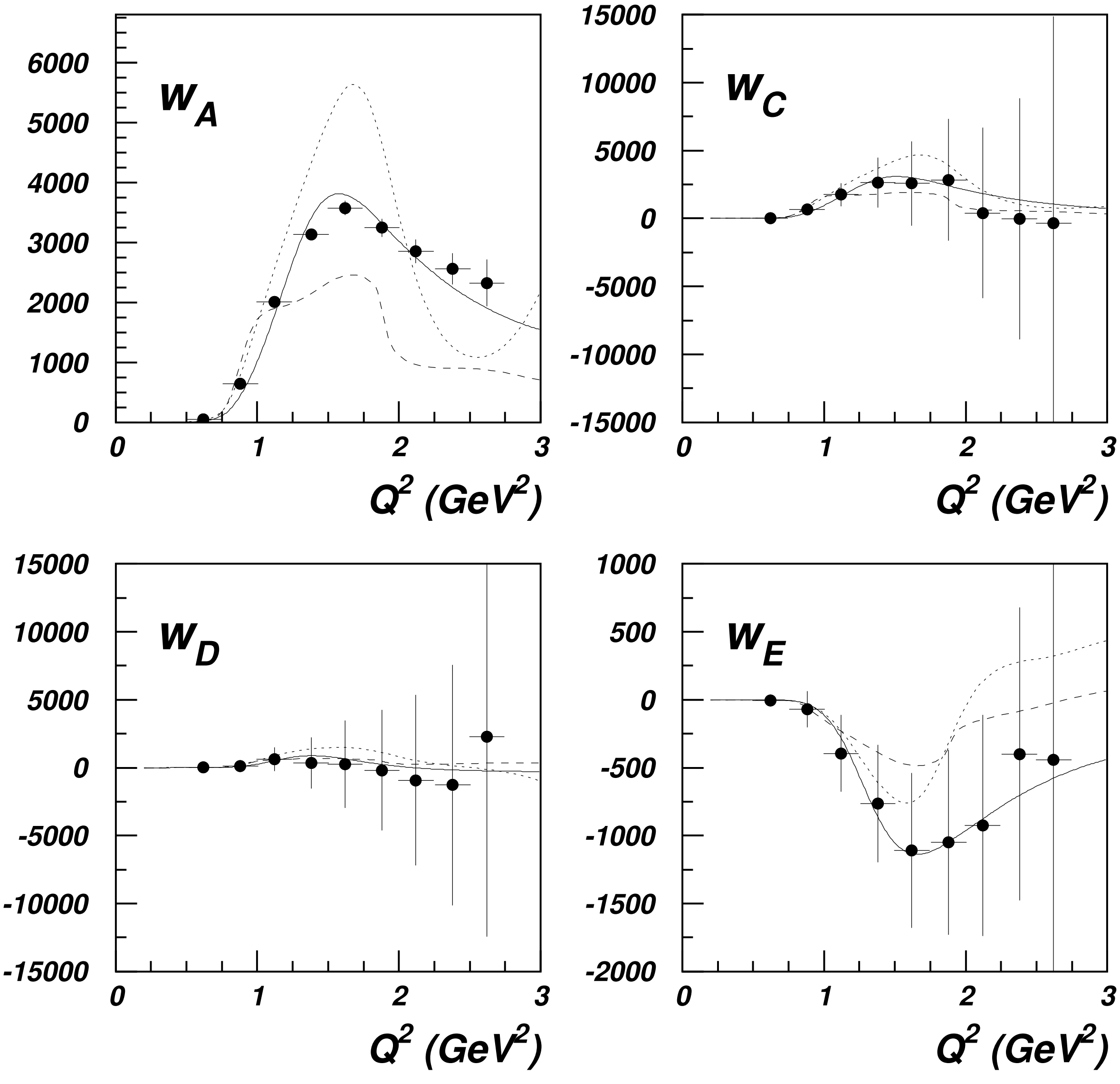


Fig. 6: Unfolding Data vs. Theory, $\gamma, \ \cos\beta, \ \cos\theta$

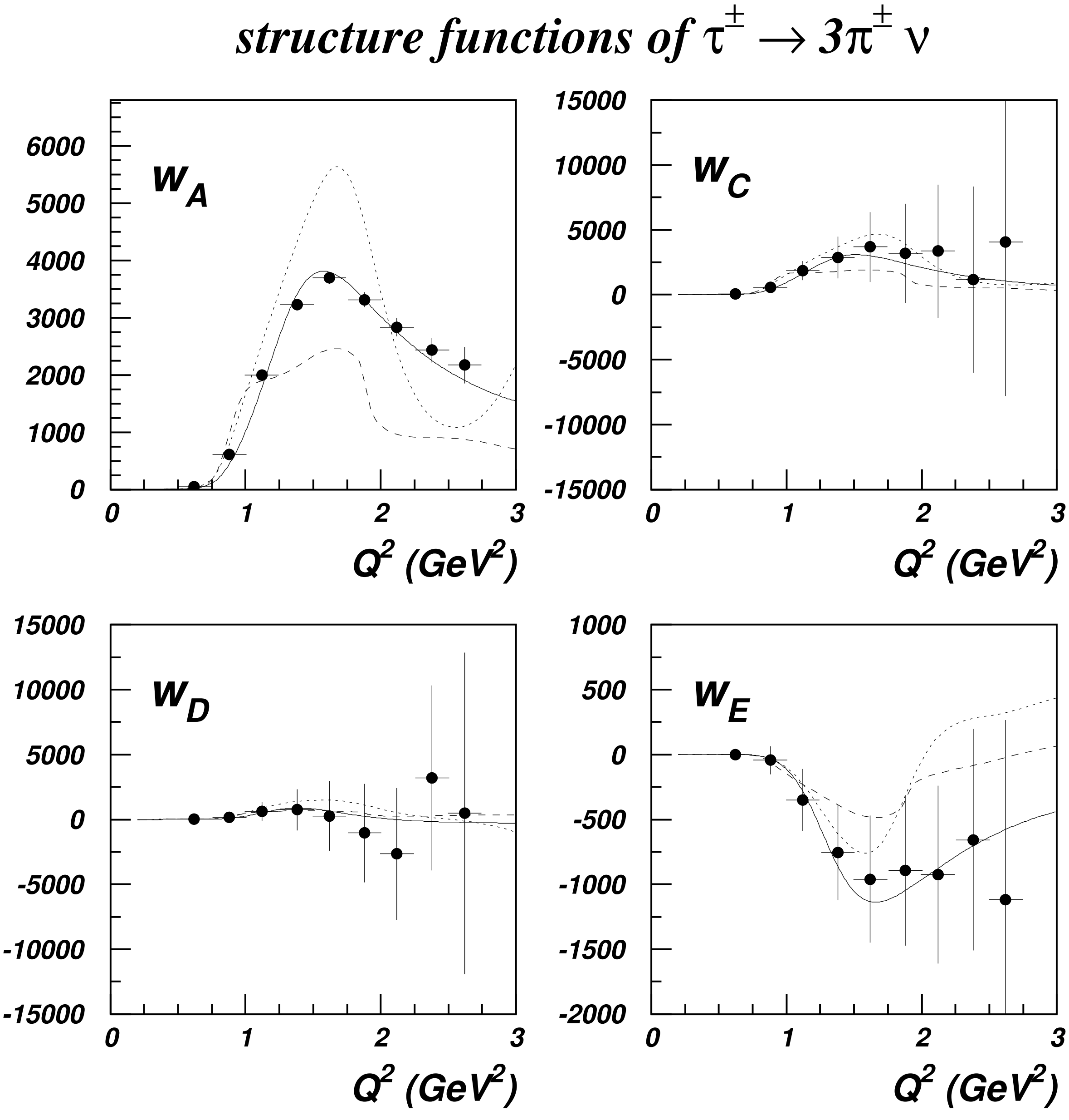
Fitted Structure Functions

Model independent analysis

structure functions of $\tau^{\pm} \rightarrow \pi^{\pm}\, 2\pi^{0}\, \nu$



Fitted Structure Functions



Fitted Structure Functions