High-precision Absolute Distance Measurement using Frequency Scanned Interferometry

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In this letter, we report high-precision absolute differential distance measurements performed with frequency scanned interferometry (FSI). Absolute differential distance was determined by counting the interference fringes produced while scanning the laser frequency. A high-finesse Fabry-Perot interferometer was used to determine frequency changes during scanning. Two new multi-distance-measurement analysis techniques were developed to improve distance precision and to extract the amplitude & frequency of vibrations present during measurements. Under laboratory conditions, a precision of 50 nm was demonstrated for a differential distance of approximately 0.7 meters.

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1. Introduction

The motivation for this project is to design a novel optical system for quasi-real time alignment of tracker detector elements used in High Energy Physics (HEP) experiments. A.F. Fox-Murphy *et.al.* from Oxford University reported their design of a frequency scanned interferometer (FSI) for precise alignment of the ATLAS Inner Detector.[1] Given the demonstrated need for improvements in detector performance, we plan to design an enhanced FSI system to be utilized for the alignment of tracker elements used in the next generation of electron positron Linear Collider (NLC) detectors. Current plans for future detectors require a spatial resolution for signals from a tracker detector such as a silicon microstrip or silicon drift detector to be approximately 7-10 μm.[2] To achieve this required spatial resolution, the measurement precision of absolute distance changes of tracker elements in one dimension should be on the order of 1 μm. Simultaneous measurements from hundreds of interferometers will be used to determine the 3-dimensional positions of the tracker elements.

We describe here a demonstration FSI system built in the laboratory for initial feasibility studies. The main goal was to determine the potential accuracy of absolute distance measurements (ADM's) that could be achieved under laboratory conditions. Secondary goals included estimating the effects of vibrations and studying error sources crucial to the absolute distance accuracy. A significant amount of research on ADM's using wavelength scanning heterodyne interferometers already exists.[3,4] In one of the more comprehensive publications on this subject, Stone *et al.* describe in detail a wavelength scanning heterodyne interferometer consisting of a system built around both a reference and a measurement interferometer.[4] In addition, this system also requires an acousto-optic modulator and polarization optics.

We believe our work represents a significant enhancement in the field of FSI in that ADM's are performed using a simple apparatus. High-precision measurements can be carried out using a tunable laser, a simple two-arm interferometer, an off-the-shelf Fabry Perot interferometer, and novel data analysis and extraction techniques. Two new multi-distance-measurement analysis techniques are presented, to improve precision and to extract the amplitude and frequency of vibrations. Major statistical and systematic uncertainties are also estimated in this letter.

2. Principles

The intensity I of any two-beam interferometer can be expressed as

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos(\phi_1 - \phi_2) \tag{1}$$

where I_1 and I_2 are the intensities of the two combined beams, ϕ_1 and ϕ_2 are the phases. Assuming the optical path lengths of the two beams are L_1 and L_2, the phase difference in Eq. (1) is $\Phi = \phi_1 - \phi_2 = 2\pi|L_1 - L_2|(\nu/c)$, where ν is the optical frequency of the laser beam, and c is the speed of light.

For a fixed path interferometer, as the frequency of the laser is continuously scanned, the optical beams will constructively and destructively interfere, causing "fringes". The number of fringes ΔN is

$$\Delta N = |L_1 - L_2|(\Delta\nu/c) = L\Delta\nu/c \tag{2}$$

where L is the optical path difference between the two beams, and $\Delta\nu$ is the scanned frequency range. The optical path difference can be determined by counting interference fringes while scanning the laser frequency. The actual differential distance can then be determined if the average refractive index over the optical path is known. In order to determine an absolute distance L_1 to a desired precision, the reference distance L_2 must be known to a better precision. Here we report only the precision of the differential distance.

3. Demonstration System of FSI

A schematic of the demonstration FSI system is shown in Fig. 1. The light source is a New Focus Velocity 6308 tunable laser (665.1 nm $< \lambda <$ 675.2 nm). A high-finesse ($>$ 200) Thorlabs SA200 Fabry-Perot Interferometer is used to measure the frequency range scanned by the laser. Data acquisition is accomplished using a National Instruments DAQ card capable of simultaneously sampling 4 channels at a rate of 5 MS/s/ch with a precision of 12-bits. Omega thermistors with a tolerance of 0.02 K and a precision of 0.01 mK were used to monitor temperature. The apparatus was supported by a damped Newport optical table.

In order to reduce air flow and temperature fluctuations, a transparent plastic box was constructed on top of the optical table. PVC pipes were installed to shield the volume of air surrounding the laser beam. The typical standard deviation (RMS) of 20 temperature measurements monitored for 20 seconds was approximately 50 mK for a thermistor placed outside of the box. Inside the PVC pipes, the typical standard deviation of 20 temperature measurements was about 0.5 mK. The interval 20 seconds is the approximate duration of the laser scan at a scan rate of 0.5 nm/s. Temperature fluctuations were suppressed by a factor of approximately 100 by employing the plastic box and PVC pipes.

4. Analysis and Results

For a FSI system, drifts and vibrations occurring along the optical path during the scan will be magnified by a factor of $\Omega = \nu/\Delta\nu$, where ν is the average optical frequency of the laser beam and $\Delta\nu$ is the scanned frequency. For the full scan of our laser, $\Omega \sim 67$. Small vibrations and drift errors that have negligible effects for many optical applications may have significant impacts on a FSI system. A single-frequency differential vibration may be expressed as $x_{vib}(t) = a_{vib}\cos(2\pi f_{vib}t + \phi_{vib})$, where a_{vib}, f_{vib} and ϕ_{vib} are the amplitude, frequency and phase of the vibration respectively. If t_0 is the start time of the scan, Eq. (2) can be re-written as

$$\Delta N = L\Delta\nu/c + 2[x_{vib}(t)\nu(t) - x_{vib}(t_0)\nu(t_0)]/c \tag{3}$$

If we approximate $\nu(t) \sim \nu(t_0) = \nu$, the measured optical path difference L_{meas} may be expressed as

$$\begin{aligned} L_{meas} = L_{true} - 4a_{vib}\Omega\sin[\pi f_{vib}(t - t_0)]\times \\ \sin[\pi f_{vib}(t + t_0) + \phi_{vib}] \end{aligned} \tag{4}$$

where L_{true} is the true optical path difference without vibration effects. If the path averaged refractive index of ambient air $\bar{n}_g$ is known, the measured absolute distance is $R_{meas} = L_{meas}/(2\bar{n}_g)$.

If the measurement window size $(t - t_0)$ is fixed and the window to measure a set of R_{meas} is sequentially shifted, the effects of the vibration will be evident. The arithmetic average of all measured R_{meas} values is taken to be the measured distance. For a large number of distance measurements N_{meas}, the vibration effects can be suppressed to a negligible level. In addition, the uncertainties from fringe and frequency determination can be decreased by a factor of ${N_{meas}}^{1/2}$ assuming they are independent for a set of distance measurements. In this way, we can improve the distance accuracy dramatically if there are no significant drift errors caused by temperature variation. This multi-distance-measurement technique is called 'slip measurement window with fixed size'. However, there is a trade off in that the thermal drift error is increased with the increase of N_{meas} because of the larger magnification factor Ω for smaller measurement window size.

In order to extract the amplitude and frequency of the vibration, another multi-distance-measurement technique called 'slip measurement window with fixed start point' was used. In Eq. (3), if t_0 is fixed, the measurement window size is enlarged for each shift. An oscillation of a set of measured R_{meas} values reflects the amplitude and frequency of vibration. This technique is not suitable for distance measurements because there always exists an initial bias term including t_0 which cannot be determined accurately in our current system.

Using the first multi-distance-measurement technique described above, the measurement window was shifted one Fabry Perot interferometer frequency peak forward for each distance measurement. The scanning rate was 0.5 nm/s and the sampling rate was 125 KS/s for all scans. 30 sequential scans were performed and recorded. Measured distances minus their average value are plotted versus number of measurements (N_{meas}) per scan in Fig. 2. It can be seen that the distance errors decrease with an increase of N_{meas}. If $N_{meas} = 1200$, the standard deviation (RMS) of absolute distance measurements for 30 scans is 49 nm. The average value of measured distances is 706451.585 μm. The relative standard deviation is $\Delta R/R_{meas} = 7.0 \times 10^{-8}$ or 70 ppb.

Using the 2nd multi-distance-measurement technique, we extracted the amplitude and frequency of the dominant vibration. A typical $N_{meas} = 200$ from one scan is shown in Fig. 3, in which every 4 adjacent distance measurements are averaged. The fitted (χ^2 minimization) amplitude and frequency of the vibration were $A_{vib} = 0.28 \pm 0.08$ μm and $f_{vib} = 2.97 \pm 0.16$ Hz, respectively with $\chi^2/n.d.f. = 22/46$.

Subsequent investigation with a CCD camera trained on the laser output revealed that the apparent 3 Hz vibration arose from the beam's centroid motion during the scan. In addition, we observed a highly reproducible 4 micron drift in measured OPD within each 20-second scan, also correlated with independently measured beam centroid drift. Because both of these effects were highly reproducible, arising we believe, from motion of the internal hinged mirror in the laser used to scan its frequency, interscan distance measurements proved highly stable, as described above. In a follow-on demonstration interferometer based on optical fiber transport from the laser to the beam splitter (to be described in a subsequent article), we observed strong suppression of the effects of beam centroid motion, as expected. We conclude that the resulting ambiguity in distance definition from beam motion on the beam splitter prevents an absolute distance determination for air transport of the beam, but that time-dependent differential distance change can be accurately determined, even with air transport.

5. Error Estimations

Some major error sources are estimated in the following;

1) Error from uncertainties of fringe and scanned frequency determination. From Eq. (2) , the measurement precision of R (the error due to the air refractive index uncertainty is considered separately below) is given by $(\sigma_R/R)^2 = (\sigma_{\Delta N}/\Delta N)^2 + (\sigma_{\Delta\nu}/\Delta\nu)^2$. For a typical scanning rate of 0.5 nm/s with 10 nm scan range, the full scan time is 20 seconds. The total number of samples for one scan is 2.5 MS if the sampling rate is 125 KS/s. There is about 3-sample ambiguity for fringe peak and valley position due to a vanishing slope and the limitation of the 12-bit sampling precision. However, there is a much smaller uncertainty for the Fabry Perot interferometer frequency peaks because of their sharpness. Thus, the estimated uncertainty is $\sigma_R/R \sim 1.27$ ppm. If $N_{meas} = 1200$, the corresponding $\Omega^* \sim 94$, $\sigma_R/R \sim 1.27$ $ppm \times \Omega^*/\Omega/1200^{1/2} \sim$ 51 ppb.

2) Error from vibrations. The detected amplitude and frequency for vibration are about 0.28 μm and 3.0 Hz. The corresponding time for $N_{meas} = 1200$ sequential measurements is 5.3 seconds. A rough estimation of the resulting error gives $\sigma_R/R \sim$ 13 ppb.

3) Error from thermal drift. The refractive index of air depends on air temperature, humidity and pressure etc. Temperature fluctuations are well controlled down to about 0.5 mK (RMS) in our laboratory by the plastic box on the optical table and the PVC pipes shielding the volume of air near the laser beam. For a room temperature of 21 ^0C, an air temperature change of 1 ^0C will result in a 0.9 ppm change of air refractive index.[5] For a temperature variation of 0.5 mK in the pipe, $N_{meas} = 1200$, the estimated error will be $\sigma_R/R \sim 0.9\ ppm/K \times 0.5\ mK \times \Omega^* \sim 42\ ppb$.

4) The air humidity variation is about 0.1% in 3 minutes, it's expected to be smaller during one scan (20 seconds). The relative error of distance $\sim$ 10 ppb. Expected fluctuations in barometric pressure should have negligible effect on distance measurements.

The total relative error from the above sources, when added in quadrature, is $\sim$ 68 ppb, with the major error sources arising from the uncertainty of fringe determination and the thermal drift. The estimated relative error agrees well with measured relative errors of 70 ppb from real data.

Besides the above error sources, other sources can contribute to systematic bias in the absolute differential distance measurement. The major systematic bias comes from uncertainty of the Free Spectral Range (FSR) of the Fabry Perot interferometer used to determine scanned frequency range precisely, the relative error would be $\sigma_R/R \sim 50\ ppb$ if the FSR was calibrated by an wavemeter with a precision of 50 ppb. A wavemeter of this precision was not available for the measurements described here. Systematic bias from uncertainties of temperature, air humidity and barometric pressure scales should have negligible effect.

6. Conclusion

A simple demonstration system of a frequency scanned interferometer was constructed to make high-precision absolute differential distance measurements. An accuracy of 50 nm for a distance of approximately 0.7 meters under laboratory conditions was achieved. Two new multi-distance-measurement analysis techniques were presented to improve absolute distance measurement and to extract the amplitude and frequency of vibrations. Major error sources were estimated, and the observed distance measurement variations were found to be in good agreement with expectation.
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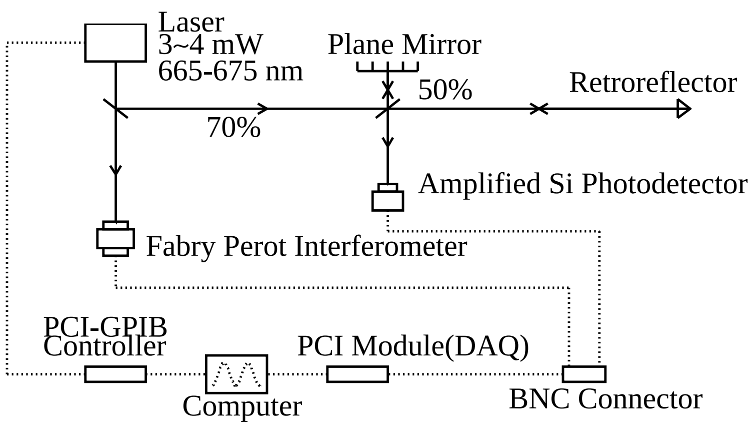

Fig. 1. Schematic of one FSI system.

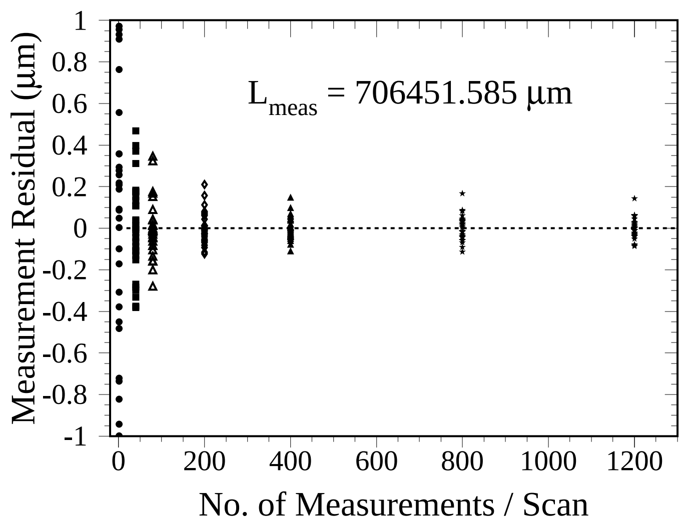

Fig. 2. The measurement residual spread of 30 sequential scans performed versus number of measurements/scan.

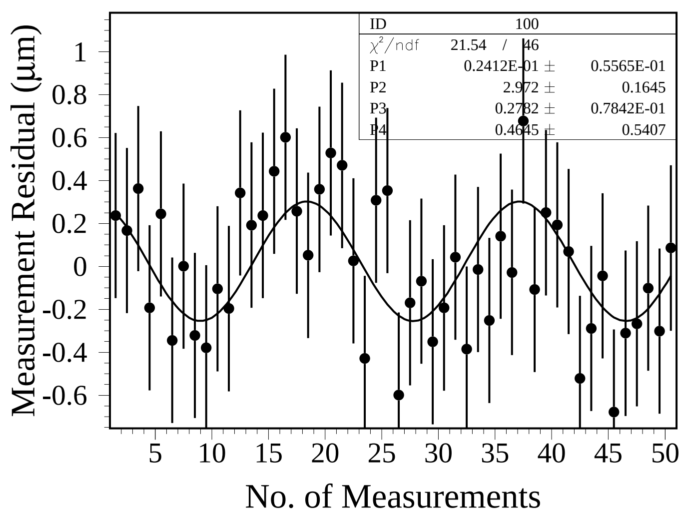

Fig. 3. Typical distribution of measurement residual. The fitted amplitude and frequency of vibration are $A_{vib} = 0.28 \pm 0.08\ \mu m$ and $f_{vib} = 2.97 \pm 0.16$ Hz respectively.