

## Supplementary Practice Problems for the Final Exam

1. An FM radio transmitter of frequency 100 MHz consists of a horizontal circular current loop of radius  $R = 5.0$  cm where the current in the loop is  $I(t) = I_0 \cos(\omega t)$  with  $I_0 = 10.0$  A.
  - (a) Find the time averaged total power radiated by the loop antenna.
  - (b) For an observer 1 km away from the antenna at the same height, what is the time averaged flux of photons coming from the antenna? Express your answer in #photons/second/cm<sup>2</sup>. Planck's constant  $h = 6.63 \times 10^{-34}$  J·s.
  
2. It was shown in lecture that for a plane electromagnetic wave incident in medium 1 upon a planar boundary with medium 2 at an incident angle  $\theta_I$  and with electric field polarization *perpendicular* to the plane of incidence, one obtains the following Fresnel's equations for the reflected and transmitted amplitudes:

$$\tilde{E}_{0R} = \left( \frac{1 - \alpha\beta}{1 + \alpha\beta} \right) \tilde{E}_{0I} \qquad \tilde{E}_{0T} = \left( \frac{2}{1 + \alpha\beta} \right) \tilde{E}_{0I}$$

where

$$\alpha \equiv \frac{\cos \theta_T}{\cos \theta_I} \qquad \beta \equiv \frac{\mu_1 v_1}{\mu_2 v_2},$$

following the same notation as used in section 9.3 of the Griffiths text.

- (a) Find the ratio  $R$  of reflected intensity to incident intensity on the planar boundary (reflection coefficient).
- (b) Find the ratio  $T$  of transmitted intensity to incident intensity on the planar boundary (transmission coefficient).
- (c) Verify explicitly that energy is conserved, *i.e.*,  $R + T = 1$ .

3. An electromagnetic wave of frequency  $6 \times 10^{14}$  Hz is incident downward in vacuum and normal onto a pool of mercury. In the following, assume that the magnetic permeability of mercury is the same as that of vacuum. Explain any approximations you make. You may wish to consult table 7.1 in the Griffiths text.
- (a) Find the reflection coefficient for this wave normally incident upon mercury from vacuum. (You may assume the dielectric constant of mercury at this frequency is comparable to one.)
  - (b) Find the skin depth of mercury for this wave frequency.
4. Thin conducting spherical shells of radii  $a$  and  $b$  ( $a < b$ ) are separated by a gap into which we can insert dielectric material. The shells are held at a potential difference  $V_0 \equiv V_a - V_b > 0$ .
- (a) If the gap is filled with a homogeneous, linear dielectric of dielectric constant  $\epsilon_r$ , find the electric field  $\vec{E}(\vec{r})$  and electric displacement  $\vec{D}(\vec{r})$  between the shells. Express your results in terms of  $V_0$ . *Show your work.*
  - (b) If the dielectric also has a non-zero conductivity  $\sigma$ , find the total current  $I$  flowing from the inner to the outer shell.
  - (c) For  $a = 1.0$  cm,  $b = 2.0$  cm,  $V_0 = 100$  V and a dielectric resistivity  $\rho = 4.6 \times 10^{-1}$   $\Omega \cdot \text{m}$ , evaluate the numerical value of the current from part (b).