

Practice Exam 2

Please budget your time in solving these problems. Make sure you attempt each problem in order to maximize partial credit. *Please show your work in your blue book.* Do not spend excessive time on any single part of a problem. There are 3 problems and 100 points possible.

You may find the following relations useful:

$$\int \frac{x^3 dx}{[x^2 + a^2]^{3/2}} = \frac{x^2 + 2a^2}{\sqrt{x^2 + a^2}}$$

and

$$\frac{1}{\sqrt{1+x}} = 1 - \frac{1}{2}x + \frac{3}{8}x^2 - \dots \quad (|x| < 1)$$

1. (45 points) The potential is given on the surface of a sphere of radius R as $V(\theta) = V_0(1 + \cos \theta)$, where V_0 is a constant.
 - (a) (18 pts) Find the potential $V(r, \theta)$ outside the sphere ($r > R$). You may leave your answer in terms of Legendre polynomials, but determine the appropriate coefficients.
 - (b) (18 pts) Find the potential inside the sphere ($r < R$).
 - (c) (9 pts) Find an expression for the surface charge density $\sigma(\theta)$ on the sphere ($r = R$).

2. (40 points) A flat compact disc (CD) of radius R with a uniform surface charge density σ and with its center at the origin lies in the x - y plane and rotates at a constant angular velocity ω about the z -axis.
 - (a) (13 pts) Find the magnetic dipole moment of the CD
 - (b) (13 pts) Find the *exact* magnetic field magnitude for a point along the positive z -axis. You may make use of the result given for example 5.6 in the Griffiths text.
 - (c) (14 pts) Show that your your answer to part (b) reduces to the expected dipole field magnitude in the limit $z \gg R$ for the dipole moment you found in part (a).

3. (15 points) Thin conducting spherical shells of radii a and b ($a < b$) are separated by a gap into which we can insert dielectric material. The shells are held at a potential difference $V_0 \equiv V_a - V_b > 0$. If the gap is filled with a homogeneous, linear dielectric of dielectric constant ϵ_r , find the electric field $\vec{E}(\vec{r})$ and electric displacement $\vec{D}(\vec{r})$ between the shells. Express your results in terms of V_0 . *Show your work.*