Practice Exam 1

Please budget your time in solving these problems. Make sure you attempt each problem in order to maximize partial credit. *Please show your work in your blue book.* Do not spend excessive time on any single part of a problem. There are 3 problems and 100 points possible.

1. (25 points)
   (a) (7 pts) Find an expression for $\vec{\nabla}\cdot[f(r)\hat{r}]$ where $f(r)$ is an arbitrary, differentiable function of r.
   (b) (6 pts) To what does your result in part a) reduce for $f(r) = r^n$?
   (c) (12 pts) Verify the divergence theorem for $n \neq -2$ by explicit volume and surface integrations. You may invoke spherical symmetry to simplify the integrals.

2. (35 points) The electric field is found to have the form $\vec{E}(\vec{r}) = k\, r^2\, \hat{r}$ for $r \leq R$ and for a constant k.
   (a) (8 pts) Find the charge density $\rho(r)$ for $r \leq R$.
   (b) (15 pts) Assuming $\rho(r) = 0$ for $r > R$, find the potential $V(r)$ outside the sphere of radius R where $V(\infty) \equiv 0$.
   (c) (12 pts) Now find the potential inside the sphere ($r < R$).

3. (40 points) Imagine a point charge q a distance z_0 above an infinite grounded plane. Define the origin to be the point on the plane closest to q, with the z-axis pointing toward q.
   (a) (10 pts) Find an expression for the potential $V(\vec{r})$.
   (b) (15 pts) The total induced charge on the plane is $-q$. Find the fraction of that induced charge that lies in a coplanar disk centered on the origin with radius $s_{\max} = z_0$.
   (c) (15 pts) Find the force on q due to this induced charge in the disk of radius z_0 and compute its ratio to the total force on q due to the entire plane's induced charge.