EXAM 2 SOLUTIONS

1. a)

\[ V_0 (1 - 3 \cos^2(\theta)) \]

BETWEEN THE SHELLS, 
\( R_1 < r < R_2 \),

\[ V = \sum_{l=0}^{\infty} \left( \frac{A_2^l}{r} + \frac{B_2^l}{r^{l+1}} \right) P_l(\cos(\theta)) \]

\[ V = 0 \]

AT \( r = R_2 \), \( V^\Pi = 0 \), OR

\[ \sum_{l=0}^{\infty} \left( A_2^l \frac{R_2^l}{R_2^{l+1}} + \frac{B_2^l}{R_2^{l+1}} \right) P_l(\cos(\theta)) = 0 \]

\[ \text{ANY } \theta \]

\[ \rightarrow B_2^l = - R_2^{l+1} A_2^l \]

\[ \rightarrow V^\Pi = \sum_{l=0}^{\infty} A_2^l \left( R_2^l - \frac{R_2^{l+1}}{l+1} \right) P_l(\cos(\theta)) \]

AT \( r = R_1 \), \( V = V_0 \left[ 1 - 3 \cos^2(\theta) \right] = -2V_0 P_2(\cos(\theta)) \)

\[ \rightarrow A_2^l = 0 \text{ FOR } l \neq 2 \]

\[ A_2^2 \left( R_1^2 - \frac{R_2^2}{R_1^2} \right) = -2V_0 \]

\[ A_2^l \left( R_1^2 - \frac{R_2^2}{R_1^2} \right) = \frac{-2V_0 R_1^3}{R_1^2 - R_2^2} = \frac{2V_0 R_1^3}{R_2^2 - R_1^2} \]
\[ V^\#(r, \theta) = \frac{2 V_0 R_1^3}{R_2^5 - R_1^5} \left( \frac{R_2^5}{r^2} - \frac{R_1^5}{r^2} \right) P_2(\cos(\theta)) \]

\[ = 2 V_0 \frac{R_2^5 - R_1^5}{R_2^5 - R_1^5} \frac{R_1^5}{r^2} \frac{1}{2} (3 \cos^2(\theta) - 1) \]

\[ = V_0 \left( \frac{R_2^5 - R_1^5}{R_2^5 - R_1^5} \right) \left( \frac{R_1^5}{r^2} \right)^2 \left[ 1 - 3 \cos^2(\theta) \right] \]

b) In region I, \[ V^+(r, \theta) = \sum_{\ell=0}^\infty \left( A^+_\ell \frac{r^2}{R_1^2} + \frac{R_2^\ell}{R_1^\ell} \right) P_\ell(\cos(\theta)) \]

At \( r = 0 \), need \( V^+ \) finite \( \Rightarrow B^+_\ell = 0 \), all \( \ell \)

At \( r = R_1 \), need \( V^+ = V_0 (1 - 3 \cos^2(\theta)) \)

\[ = -2 V_0 P_2(\cos(\theta)) \]

\[ \Rightarrow \sum_{\ell=0}^\infty A^+_\ell \frac{R_1^2}{R_1^2} P_\ell(\cos(\theta)) = -2 V_0 P_2(\cos(\theta)) \]

\[ \Rightarrow A^+_\ell = 0 \quad \text{for} \quad \ell \neq 2 \]

\[ A^+_2 = -2 \frac{V_0}{R_1^2} \]

\[ \Rightarrow V^+(r, \theta) = -2 \frac{V_0}{R_1^2} \frac{1}{2} \left( 3 \cos^2(\theta) - 1 \right) \]

\[ = V_0 \left( \frac{r}{R_1} \right)^2 \left( 1 - 3 \cos^2(\theta) \right) \]
a) AMPERIAN LOOP

AMPERE'S LAW: \[ \mathbf{B}_{\text{IN}}(s) \cdot 2\pi s = M_0 \oint ds' \left< \mathbf{J}^i(s') \right> (\mathbf{k}s) \]

\[ \mathbf{j}(s) = k s \hat{z} \]
\[ \mathbf{B}_{\text{IN}}^i(s) = \mathbf{B}_{\text{IN}}(s) \cdot \hat{z} \]
\[ \mathbf{B}_{\text{IN}}(s) = \frac{M_0 k s^2}{3} \]

\[ \mathbf{j}(s) = M_0 k s^2 \]

\[ \mathbf{B}_{\text{IN}}(s) = \frac{2 M_0 k s^2}{3} \]

TOTAL CURRENT IN WIRE IS \[ I = \oint ds' \int_0^{2\pi} \mathbf{J}^i(s') ds' = \frac{2\pi k a^2}{3} \]

\[ I = \frac{3 I}{2\pi a^3} \]

OR \[ \mathbf{B}_{\text{IN}}(s) = \frac{M_0 (3 I)}{3 (2\pi a^3)} \]

b) OUTSIDE THE WIRE,

\[ \mathbf{B}_{\text{OUT}}(s) \cdot 2\pi S = \frac{M_0 I}{2\pi a} \]

\[ \mathbf{B}_{\text{OUT}}(s) = \frac{M_0 I}{2\pi S} \]

\[ \mathbf{B}_{\text{OUT}}(s) = \frac{M_0 I}{2\pi a} \]

\[ \mathbf{A}_{\text{IN}} \times \mathbf{A}_{\text{IN}} = \mathbf{B}_{\text{IN}} \]

\[ \mathbf{A}_{\text{IN}} \times \mathbf{A}_{\text{IN}} = (\mathbf{A}_{\text{IN}} \times \mathbf{A}_{\text{IN}}) \cdot \mathbf{B}_{\text{IN}}(s) \]

\[ \frac{\partial A_z}{\partial z} - \frac{\partial A_z}{\partial S} = \frac{M_0 I}{2\pi a^3} s^2 \]

\[ \frac{\partial A_z}{\partial S} = -\frac{M_0 I}{2\pi a^3} s^2 \]
\[ \Rightarrow A_z = -\frac{M_0 I}{6\pi a^2} s^3 + C \]

\[ A_z (s=a) = 0 \Rightarrow C = \frac{M_0 I}{6\pi} \]

\[ \Rightarrow A_{IN} = \frac{M_0 I}{6\pi} \left(1 - \frac{s^3}{a^3} \right) \frac{2}{3} \]
\( a \)) \( \vec{p} = Kr \)

\[ \sigma_{\text{surf}} = \vec{p} \cdot \hat{n} \]

**WHERE** \( \hat{n} = \hat{z} \)

**AND** \( \rho_z = (Kz)^{2/a} = \frac{K^2}{3} \)

\( b \)) \( \rho_\phi = -\nabla \cdot \vec{p} = -K \nabla \cdot \vec{p} = -3K \)

\( c \)) **TOTAL SURFACE CHARGE (BY SYMMETRY)**

\[ 6 \left( K \left( \frac{a^2}{2} \right) \right) = 3K a^3 \]

**TOTAL VOLUME CHARGE IS**

\[ (-3K) \left( \frac{a^3}{2} \right) = -3K a^3 \]

\[ \rightarrow \text{NET CHARGE} = 3K a^3 - 3K a^3 = 0 \]

\( d \)) **THE SURFACE CHARGE DENSITY ON THE CAVITY SURFACE IS**

\[ \vec{p} \cdot \hat{n} = \left( (Kr)^2 \right) \frac{\hat{z}}{2} |_{r=b} \]

\[ \text{CAVITY} = -Kb \]

**AND THE TOTAL SURFACE CHARGE IS**

\[ (-Kb) \left( 4\pi b^2 \right) = -4\pi Kb^3 \]

**THE VOLUME CHARGE IS**

\[ (-3K) \left( a^3 - \frac{4\pi b^3}{3} \right) = -3K a^3 + 4\pi Kb^3 \]

\[ \text{OR} \quad \text{NET CHARGE} = 3K a^3 - 4\pi Kb^3 - 3K a^3 + 4\pi Kb^3 \]

\[ = 0 \]