Physics 405 – Fall 2008

Exam 2

October 31, 2008

Please budget your time in solving these problems. Make sure you attempt each part of every problem. If you lack a necessary result from a previous part of the problem, explain how you would use the missing result, in order to maximize partial credit. *Please show your work in your blue book.* Do not spend excessive time on any single part of a problem. There are 3 problems and 100 points possible.

1. (48 points) The potential is given on the surface of a spherical shell centered on the origin with radius R_1 as $V(R_1, \theta) = V_0 \left[1 - 3\cos^2(\theta)\right]$ where V_0 is a constant. Concentric with this spherical surface is a grounded conducting sphere of radius $R_2 > R_1$. Denote region I as having $r < R_1$ and region II as having $R_1 < r < R_2$.

   (a) (32 pts) Find the potential $V^{\mathrm{II}}(r, \theta)$ in region II. Express your result in terms of V_0, R_1, and R_2.

   (b) (16 pts) Find the potential $V^{\mathrm{I}}(r, \theta)$ in region I. Express your result in terms of V_0 and R_1.

2. (28 points) A steady current I flows down a long straight cylindrical wire of radius a in such a way that the current density $\vec{J}$ has a magnitude proportional to s where s is the distance from the wire's central axis (define to be the z axis).

   (a) (13 pts) Find the magnetic field $\vec{B}_{\mathrm{IN}}$ inside the wire ($s < a$). Express your result in terms of the total current I.

   (b) (6 pts) Find the magnetic field $\vec{B}_{\mathrm{OUT}}$ outside the wire ($s > a$). Express your result in terms of the total current I. Verify consistency of your results for parts (a) and (b) at $s = a$.

   (c) (9 pts) From the magnetic field obtained in part (a), obtain the magnetic vector potential $\vec{A}$ inside the wire, subject to the arbitrary constraint $\vec{A}(s = a) = 0$. You may assume that $\vec{A}$ has no dependence on z.

3. (24 points) A dielectric cube of side a, centered at the origin, carries a "frozen-in" polarization $\vec{P} = k\vec{r}$ where k is a constant.

   (a) (5 pts) Find the bound surface charge density σ_b on the top face of the cube.

   (b) (5 pts) Find the bound volume charge density ρ_b inside the cube.

   (c) (6 pts) Verify that the sum of all bound charges for the cube is zero.

   (d) (8 pts) Now consider the same polarized cube but with a spherical cavity of radius $b < a/2$ centered on the origin. Repeat part (c) for this case.