Supplemental Practice Exam Problems

1. Two identical blocks of mass m are constrained to move in one dimension (horizontal) on frictionless ice and are attached to one another via a spring of spring constant κ, as shown below.

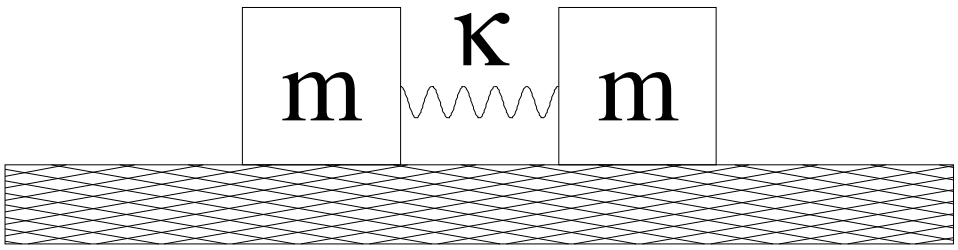
   (a) Define x_1 and x_2 as the horizontal displacements of the masses from their equilibrium points when the system center of mass is at rest. Write down the Lagrangian of this system.

   (b) Use the Lagrangian to derive the two equations of motion.

   (c) Find the eigenfrequencies of this coupled system.

   (d) Describe qualitatively the normal modes of this system.

   (e) Compare these eigenfrequencies to the values obtained for each mass when the other mass is held fixed.



2. A symmetric gyroscopic top of mass M with a ball tip rests in a stationary cup, allowing the top's symmetry axis to lie horizontal. Assume the top's moment of inertia about its symmetry axis is I_3 and its other moments of inertia w.r.t. the tip are both I_1, where the center of mass of the top is a distance h from the tip. The top is given an angular velocity magnitude $\omega_3 > 0$ about its symmetry axis.

   (a) Using the analysis method of section 11.11 of the text, find a precession rate $\dot{\phi}$ for which the top's motion is pure precession in a horizontal plane (without nutation).

   (b) Show that the top's motion is stable with respect to perturbations in θ, *i.e.*,, if jostled up or down slightly, the top feels a restoring torque back toward the horizontal.