Practice Exam 1

Please budget your time in solving these problems. Make sure you attempt each problem in order to maximize partial credit. Please show your work in your blue book. Do not spend excessive time on any single part of a problem. There are 3 problems and 100 points possible.

1. (30 points) A projectile is fired from the origin of the coordinate system at $t = 0$ (x horizontal, y vertical) with an initial speed $v_0$ and in a direction in the vertical $x$-$y$ plane making an angle $\alpha$ with the horizontal.

   (a) (15 pts) Calculate the time required for the projectile to land on a parabolic hillside described by $y = \beta x^2$ with $\beta > 0$. Ignore air resistance.

   (b) (10 pts) Find the maximum angle $\alpha$ (fixed $v_0, \beta$) for which the projectile has not yet reached the peak of its trajectory when it hits the hillside.

   (c) (5 pts) Find the minimum $v_0$ (fixed $\beta$) required for there to be a physical solution to part b).

\[ y(x) = \beta x^2 \]
2. (40 points) Two identical blocks, each of mass $m$, are glued together end-to-end (with bad, massless glue) and hang at rest in equilibrium from a massless spring of spring constant $k$, as shown. The other end of the spring is attached to the ceiling. Define the origin of the coordinate system to be this point of attachment and define the $y$-axis to be positive downward. The length of the spring in its fully relaxed state is $\ell$. At time $t = 0$, the glue suddenly gives way, and the bottom block falls to the floor.

(a) (32 pts) Ignoring any friction, find the position $y(t)$ of the top of the top block for $t > 0$.

(b) (8 pts) Now assume there is a frictional force $F_{\text{frict}} = -by$ opposing the top block’s vertical motion ($b$ is a real, positive constant). How much total energy is dissipated in friction as the top block settles to its final equilibrium? Think carefully before calculating.
3. (30 points) *Using the technique of Lagrange multipliers* (as in section 6.6 of the text), show that for a right parallelepiped of fixed volume $V$, the total surface area of the six sides is minimized if the parallelepiped is a cube. (A solution not based on Lagrange multipliers will earn no more than $2/3$ of full credit for this problem.)