Coherence of Power Lines

1 Introduction

Using the QMLR Line Monitor the parameters (amplitude $a(t)$ and phase $Y(t)$) of the LIGO power monitor (PM) signals were measured

$$s(t) = a(t) \cos(Y(t)).$$

In case of two power monitors (PM) s_L (LLO) and s_H (LHO) the power coherence between Livingston and Hanford was studied.

2 Parameters of 60Hz signal

The parameters of power monitor signals were measured using data segments of one second long. For each data segment d_i, the average amplitude a_i and the phase Ψ_i were measured. The power frequency was estimated as a derivative of the phase Ψ(t). Approximately 1 hour of data, starting at UTC time 668212508 was used in this analysis.

2.1 Amplitude

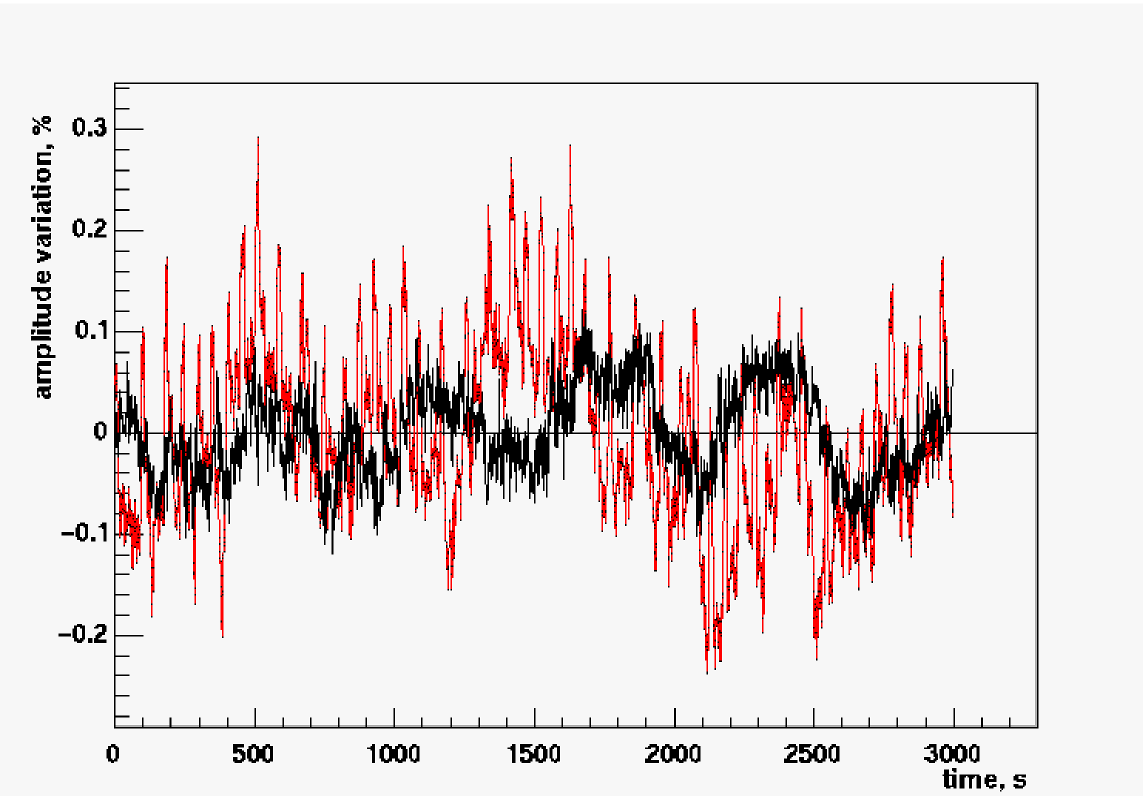

Figure 1. Amplitude variation: L0:PEM-LVEA_V1 (black) and H0:PEM-LVEA2_V1 (red)

2.2 Phase

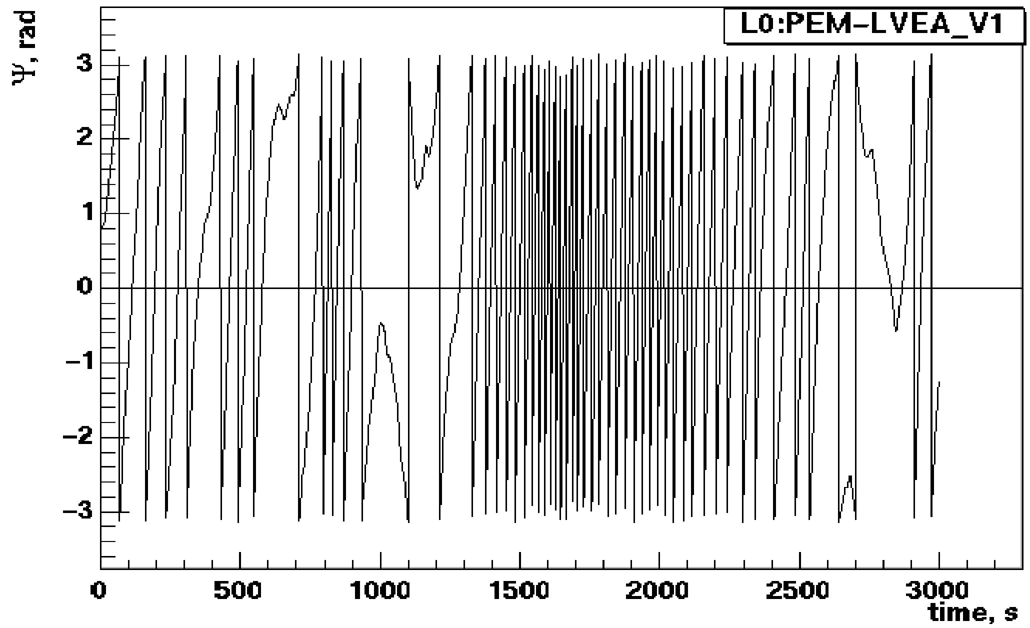

Figure 2. Phase Ψ(t) for the L0:PEM-LVEA_V1 channel.

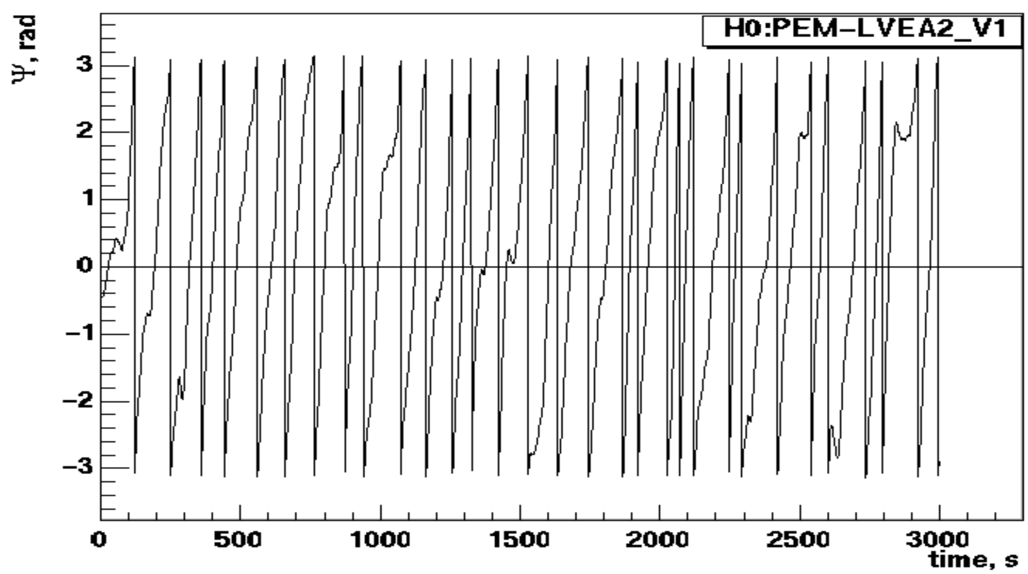

Figure 3. Phase Ψ(t) for the H0:PEM-LVEA2_V1 channel.

2.3 Frequency

During one second time interval the power frequency doesn't change much. The Line Monitor measures average frequency, which is used as an estimate of instantaneous power frequency. The measured frequency may vary with time as shown in Figure 4.

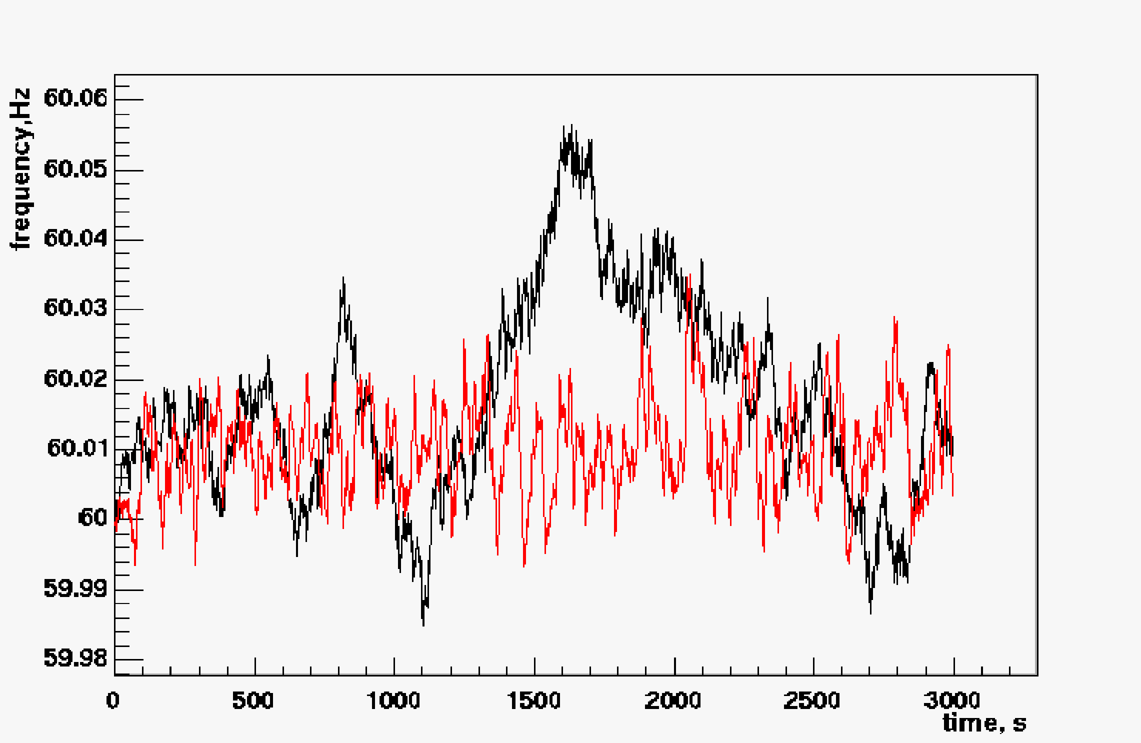

Figure 4. Power frequency for the L0:PEM-LVEA_V1 (black) and H0:PEM-LVEA2_V1 (red) channels

3 Coherence of 60 Hz lines

A sum of two harmonic oscillations $s_L(t)$ and $s_H(t)$ with the same frequency is also a harmonic oscillation

$$s(t) = s_L(t) + s_H(t) = A \cdot \sin(wt + q),$$

where the amplitude A is given by the following equation

$$A^2 = a_L^2 + a_H^2 + 2a_L a_H \cos(f_L - f_H),$$

and the average (over the time interval T) square amplitude is[1]

$$\overline{A}^2 = a_L^2 + a_H^2 + 2a_L a_H \frac{1}{T}\int \cos(f_L - f_H)dt = a_L^2 + a_H^2 + 2a_L a_H \overline{\cos(\Delta f)}.$$

If the phase difference remains constant during time T, the signals $s_L(t)$ and $s_H(t)$ are coherent. In case, if the phase difference Δf changes randomly in time and the observation time T_{tot} is long enough, the $s_L(t)$ and $s_H(t)$ are not coherent and the interference term in the last equation is zero.

For coherence study approximately 10 hours of the E3 run data starting at the UTC time 668212508 was analyzed. Figure 5 shows the average $\cos(\Delta f)$ (T=1sec) for the L0:PEM-LVEA_V1 and L0:PEM-LVEA2_V1 channels as a function of time. One can see that there are sections of data when two power monitors are quite coherent. However, for a long run (10 hours) the $\Delta\phi$ distribution is close to uniform (see Figure 6).

To characterize the $\Delta\phi$ uniformity the *coherence coefficient* γ is used

$$g = \tfrac{1}{N}\left|\sum_{k=1}^{N} \exp(i\Delta f_k)\right|, \quad N = T_{tot}/T.$$

In our case γ=0.031, what is consistent with the uniform phase distribution (significance level SL=16%). The average coherence time (T_c) is 38 seconds.

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[1] The amplitudes a_H and a_L are assumed to be constant.

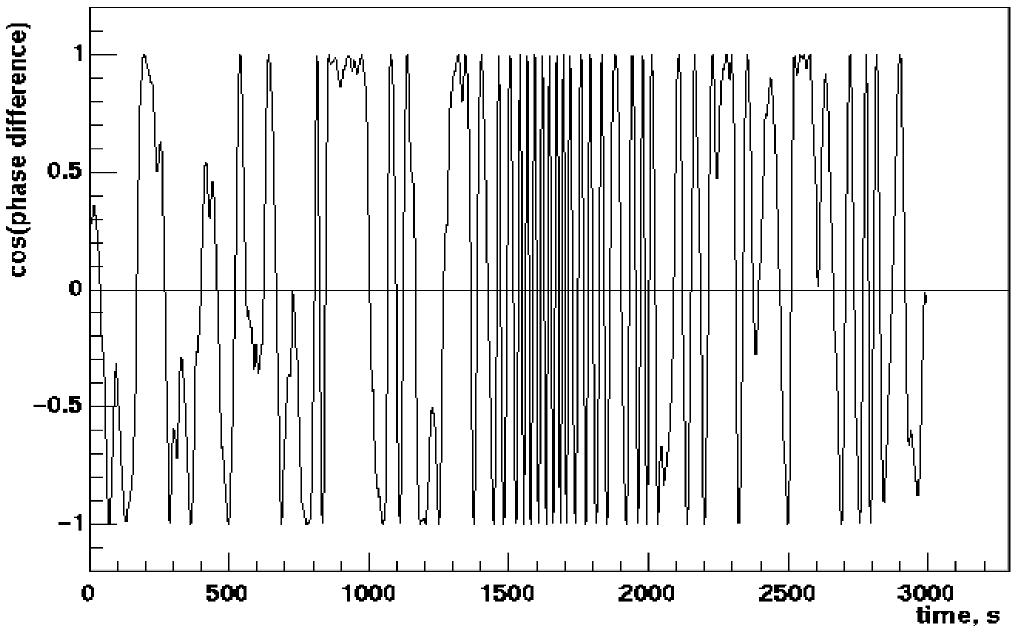

Figure 5.Interference term variation ($\cos(\Delta f)$) as a function of time.

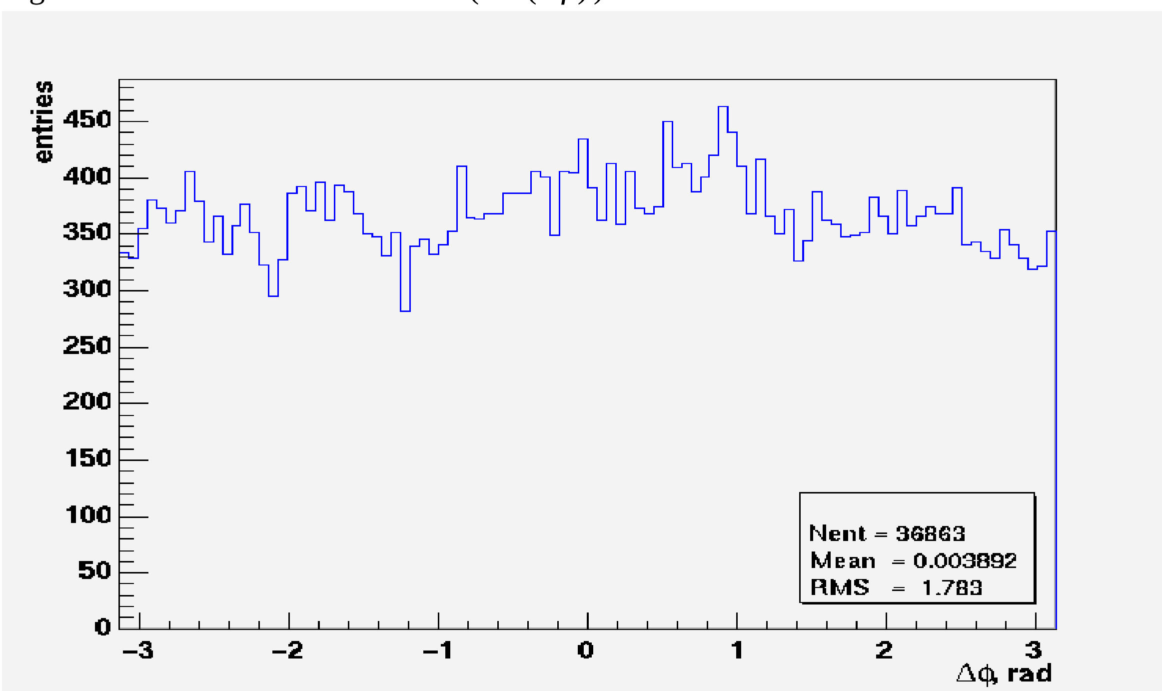

Figure 6. Phase difference between the H0:PEM-LVEA2_V1 and L0:PEM-LVEA_V1.

In a similar way the coherence of signals $s_L(t)$ and $s_H(t+\tau)$ can be calculated, where τ is a time delay between two signals. The coherence coefficient γ as a function of τ is shown on Figure 7. Figure 8 shows the significance level, which is a probability to yield measured coherence coefficient assuming the uniform distribution of $\Delta\phi$. Note that for τ=-340s and τ=560s the coherence coefficient γ is around 0.07 and significance level is much less then 0.1%. It is an indication of presence of a small degree of coherence between LLO and LHO power mains. Figure 9 shows the phase distributions at this τ, which are quite non-uniform.

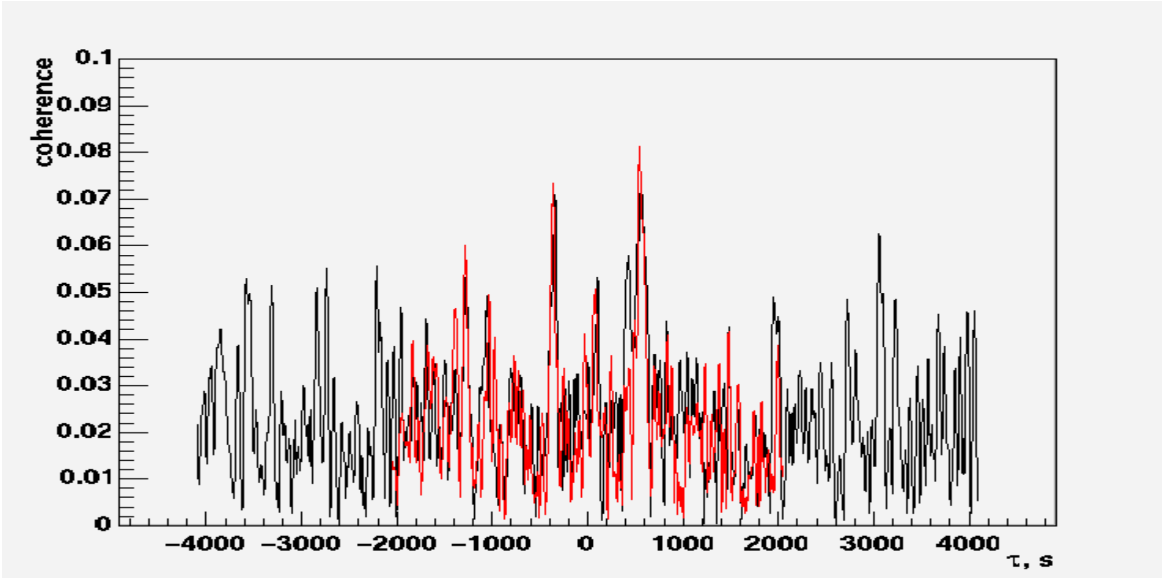

Figure 7. Coherence as a function of the τ: power monitors (black), magnetometers (red).

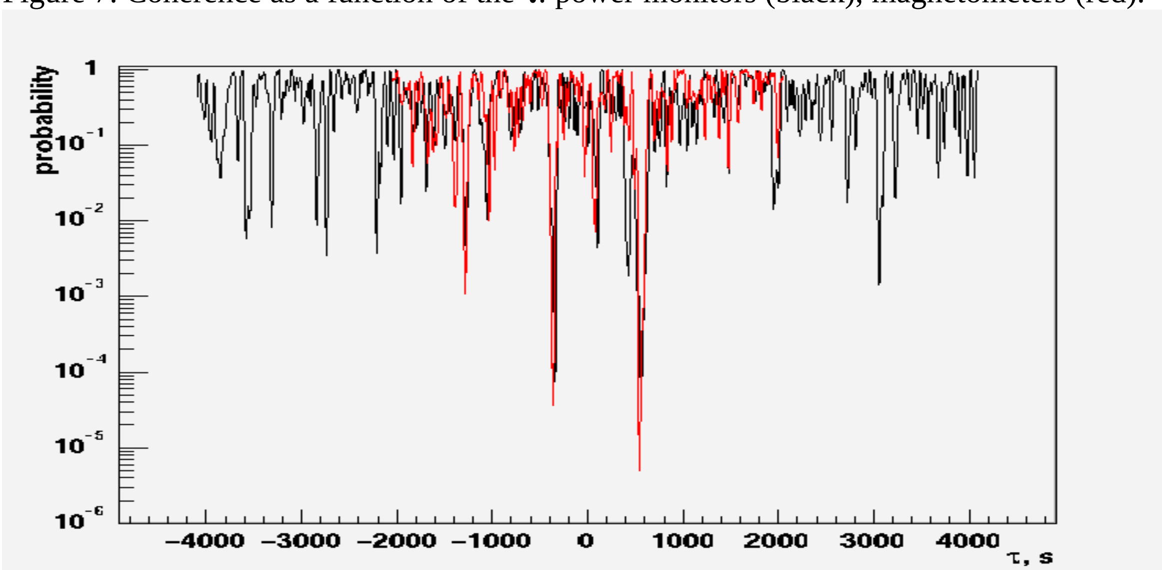

Figure 8. Significance level as a function of τ; power monitors (blk), magnetometers (red)

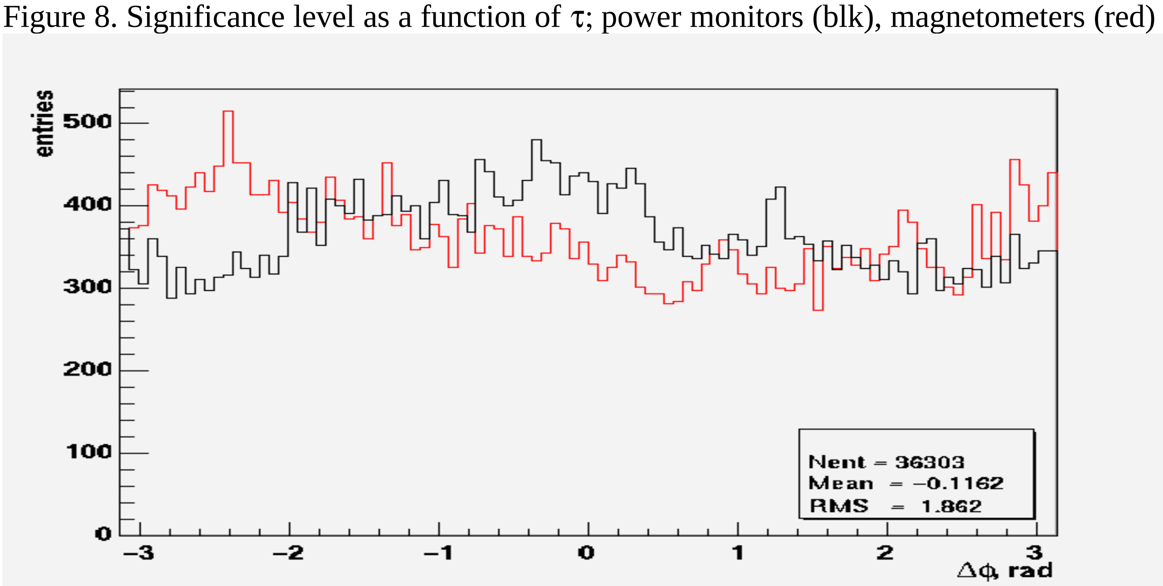

Figure 9. Phase difference distribution for PM, τ=-341sec (black) and τ=560sec (red)

The power lines are detected by the magnetometers as well. They show results almost identical with the power monitors (Figures 9,8; red curves).

For comparison, Figures 10, 11 show the $\Delta\phi$ distribution for the L0:PEM-LVEA_V1 and L0:PEM-EX_V1 channels. Those two signals are very coherent (γ~1.) as it should be for the same power system.

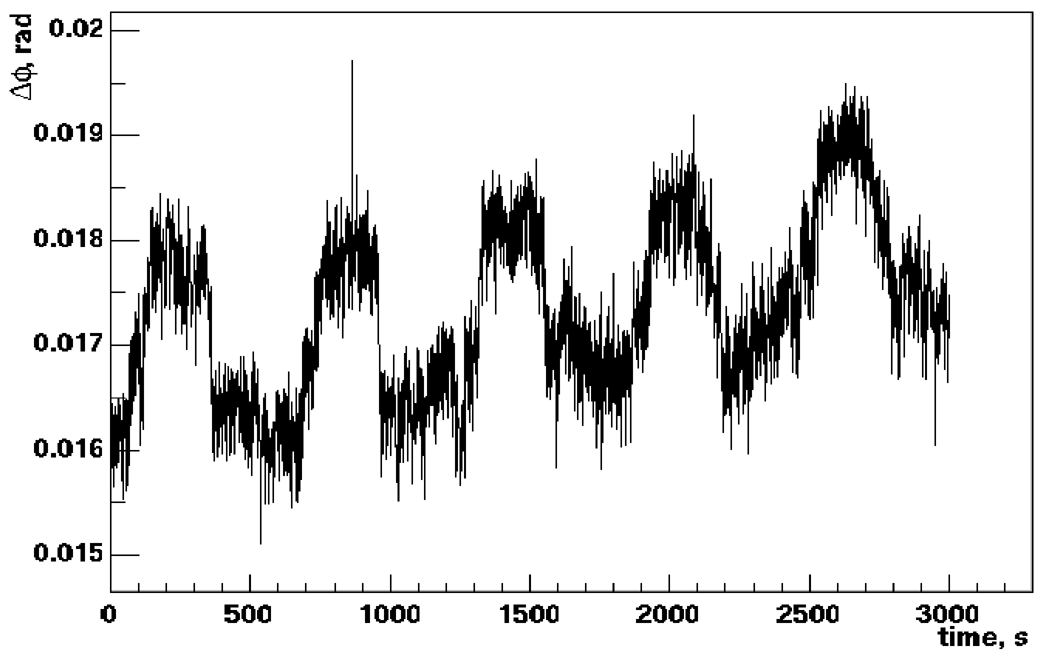

Figure 10. Phase difference between the L0:PEM-EX_V1 and L0:PEM-LVEA_V1 as a function of time

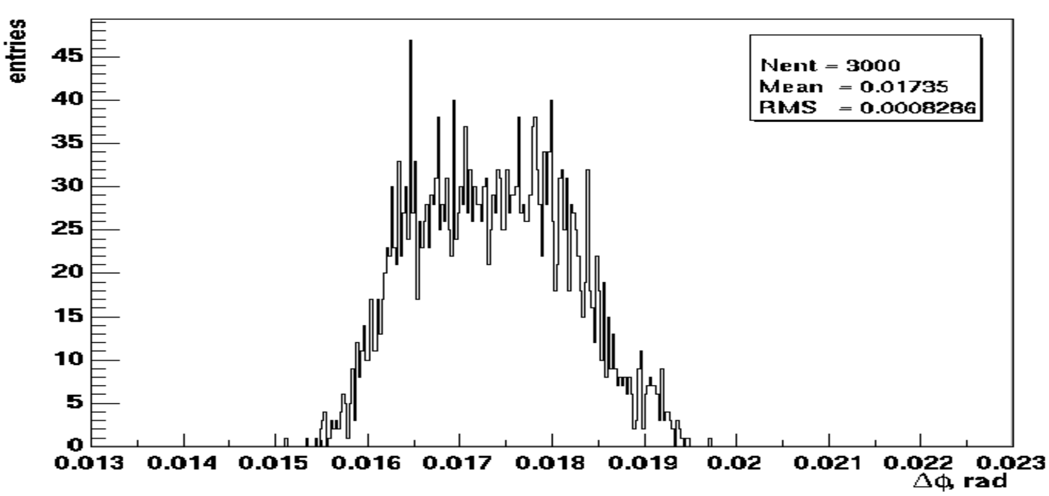

Figure 11. Phase difference between the L0:PEM-EX_V1 and L0:PEM-LVEA_V1 as a function of time

3.1 Interpretation

One possible explanation of the observed coherence between the LLO and LHO power monitors is the following. In ideal case the phase of each monitor is a linear function of time: $f = w_0 t + const$. Then the monitors are perfectly coherent. In real life there is an additional (hopefully random) phase $j(t)$, so $f = w_0 t + j(t) + const$. Lets assume the phase j has a harmonic term $r\cos(nt + q)$ and the frequency n is the same for both sites

$$j(t) = r\cos(nt + q) + h(t),$$

where h is a random phase. Then the phase difference between sites would be

$$\Delta f = r_L \cos(nt + q_L) - r_H \cos(nt + q_H) + h_L(t) - h_H(t).$$

If the h term is small, the coherence is determined by the phase difference $q_L - q_H$ .In this case we would expect to see equidistant peaks in the $\gamma(\tau)$ curve. This picture would be more complicated if there are several modulation frequencies n . This model agrees with what we see in the Fourier spectra of the phase difference (Figure 12), which shows that Δf has a modulation at frequencies around 0.5, 1., 2. mHz.

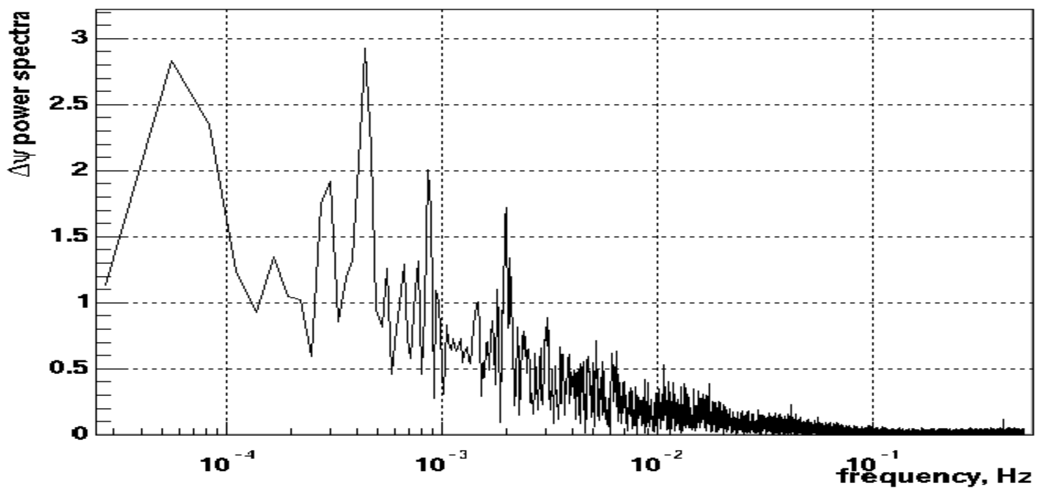

Figure 12. Δf Fourier spectra.

4 Conclusion

Although, there is some indication of power coherence between the LHO and LLO sites for this particular interval of time, they may not be coherent in a longer run. To conclude if there are periods of time when the LLO-LHO coherence time is much longer then 1 minute and there is a certain degree of coherence between sites, 24 hours of data for different days of week should be analyzed.