

Search for Neutrino Oscillation with MiniBooNE Detector

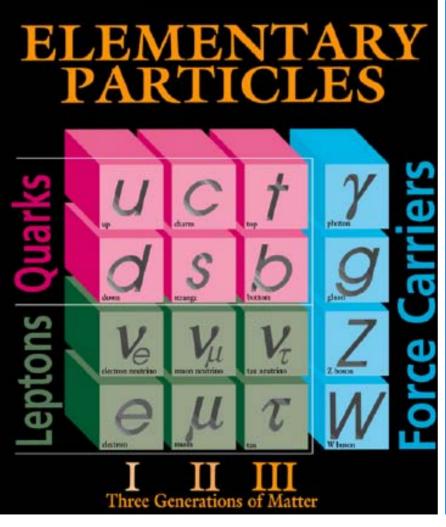
Hai-Jun Yang University of Michigan

University of Nebraska Lincoln, Nov. 29, 2007

Outline

- Brief introduction of neutrino
- Physics Motivation of MiniBooNE
- MiniBooNE Neutrino Beam
- Events in the Detector
- Two Independent Analyses
- MiniBooNE Results

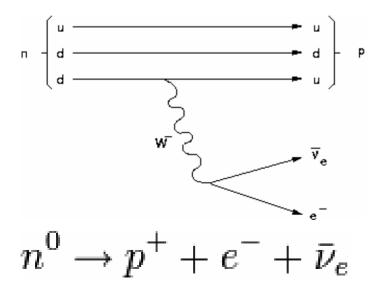
The Standard Model

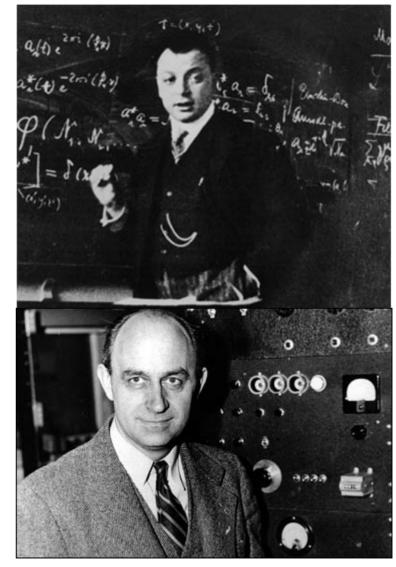


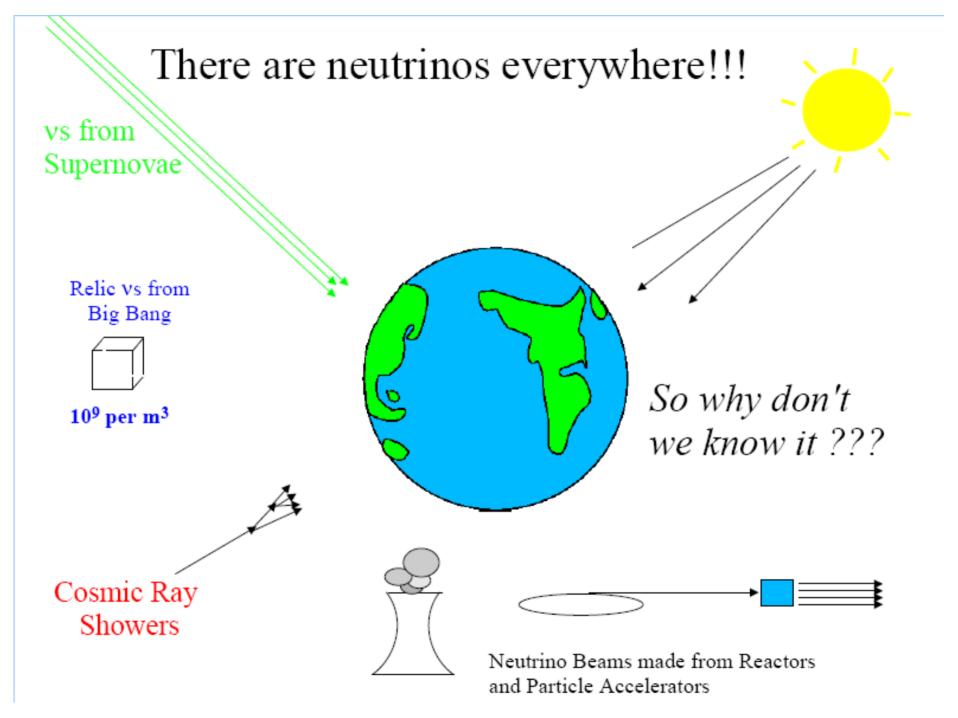
1900s:	e discovered (cathode ray tube)
	γ interpreted as a particle
1930s:	μ discovered (cosmic rays)
1950s:	v_{e} observed (nuclear reactor)
	v_{μ} discovered (BNL)
1960s:	1 st evidence for quarks
	u and d observed (SLAC)
	s observed (BNL)
1970s:	standard model is born
	c discovered (SLAC, BNL)
	τ observed (SLAC)
	b observed (FNAL)
1980s:	W and Z observed (CERN)
1990s:	t quark observed (FNAL)
2000s:	v_{τ} observed (FNAL)

About Neutrino

- Wolfgang Pauli postulated existence of neutrino ("little neutral ones") in order to explain the missing energy in nuclear β^- decay in 1930.
- Enrico Fermi presented theory of beta decay in 1934.

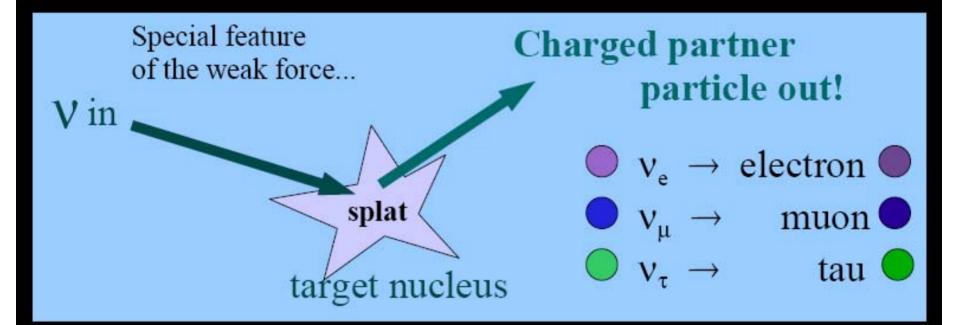






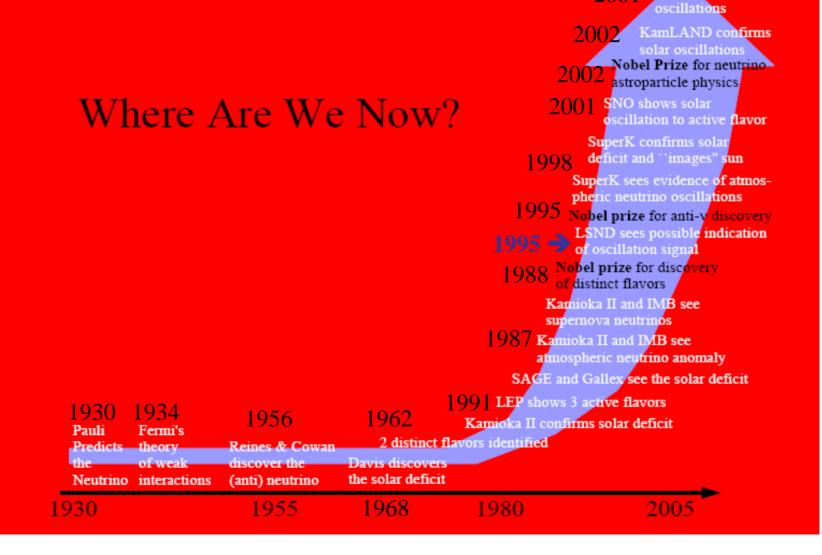
Detecting Neutrinos

Seeing *neutral* particles is <u>really</u> hard, but when vs interact via the ``Charged Current Interaction," a v goes in, and its *charged* partner particle comes out



...by observing the charged particle partner, one can **infer** the neutrino flavor

Brief History of Neutrino

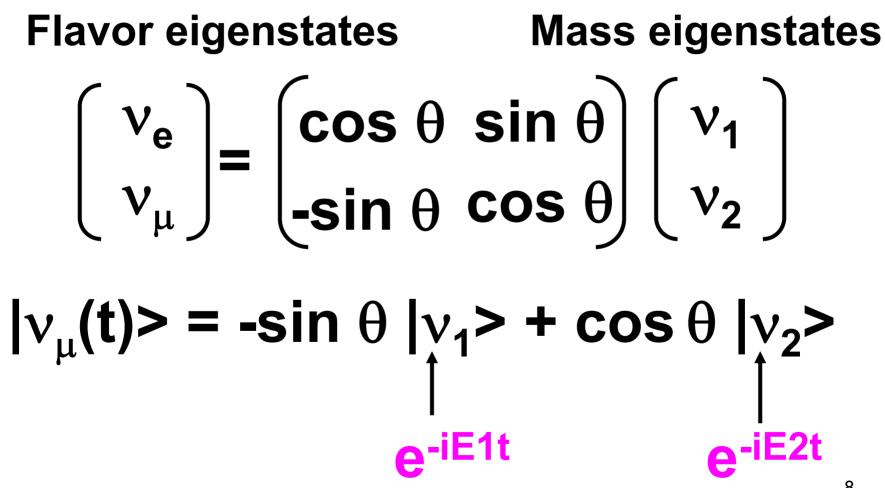


K2K confirms atmospheric

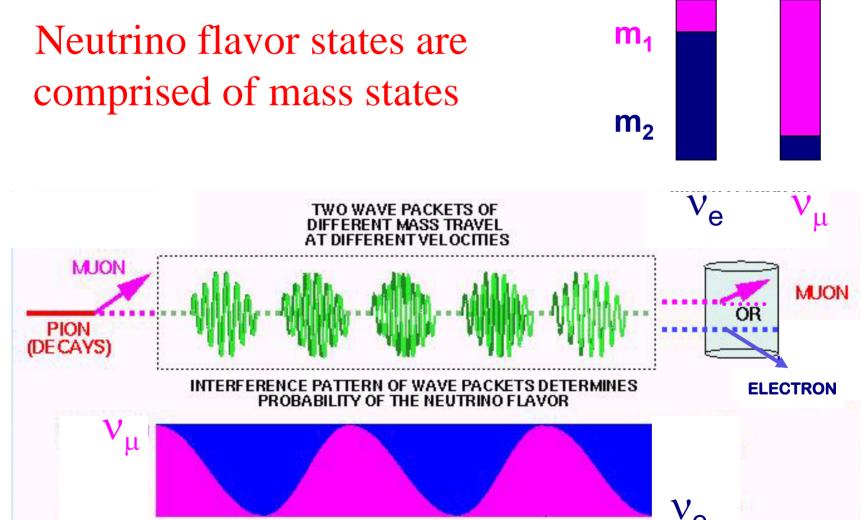
2001

Neutrino Oscillations (2 flavors)

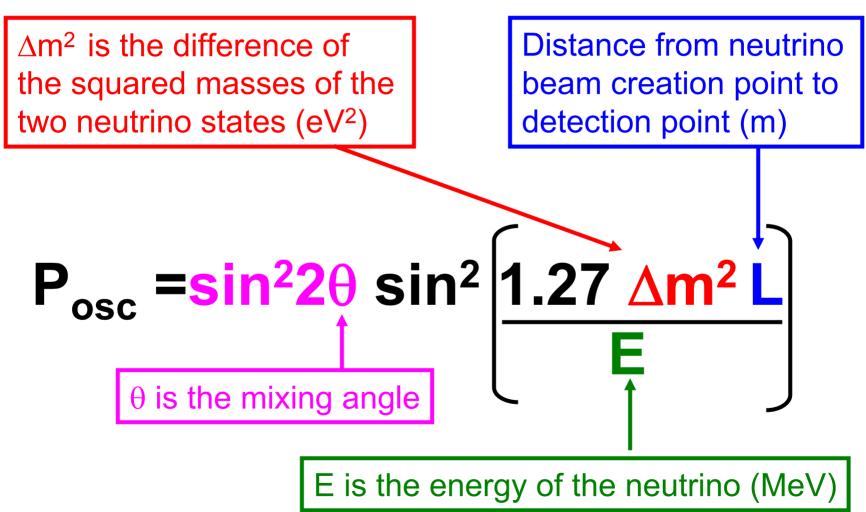
(For 3 v flavors mixing, it needs 3×3 unitary matrix with CP-violating phase.)



Neutrino Oscillations (2 flavors)



Neutrino Oscillations (2 flavors)



Neutrino Oscillation Parameters

Solar Neutrino Oscillation (Homestake, GALLEX, SAGE, Kamiokande-II, Super-K, SNO etc.), confirmed by KamLAND (reactor beam)

$$\Delta m_{21}^2 = \Delta m_{\text{sol}}^2 = 8.0^{+0.6}_{-0.4} \cdot 10^{-5} \text{eV}^2$$
$$\tan^2(\theta_{12}) = 0.45^{+0.09}_{-0.07} \qquad \theta_{12} = \theta_{\text{sol}} = 33.9^{\circ} {}^{+2.4^{\circ}}_{-2.2^{\circ}}$$

Atmospheric Neutrino Oscillation (IMB,MARCO,Soudan, Kamiokande-II, Super-K etc.), confirmed by K2K, MINOS (accelerator beam)

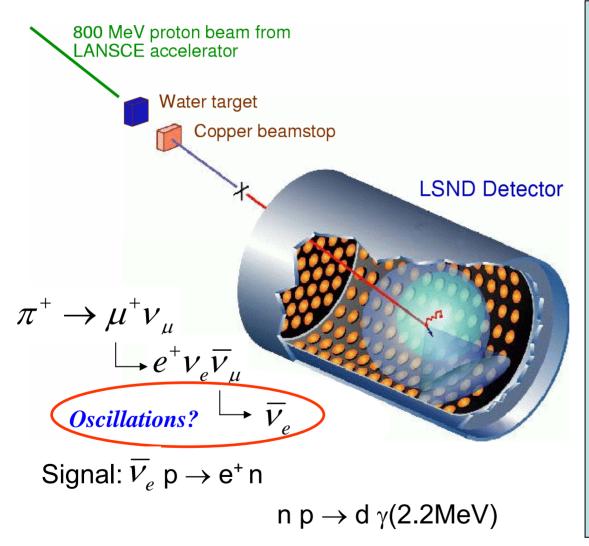
$$\Delta m_{31}^2 \approx \Delta m_{32}^2 = \Delta m_{\text{atm}}^2 = 2.4^{+0.6}_{-0.5} \cdot 10^{-3} \text{eV}^2$$
$$\sin^2(2\theta_{23}) = 1^{+0}_{-0.1} \qquad \theta_{23} = \theta_{\text{atm}} = 45 \pm 7^\circ$$

→Chooz (reactor beam)

future exp., Double Chooz, Daya Bay(reactor), NOvA, T2K(accelerator)

$$\sin^2(2 heta_{13}) < 0.12$$
 (10°)

The LSND Experiment



LSND took data from 1993-98

Nearly 49000 Coulombs of protons on target

Baseline: 30 meters

Neutrino Energy: 20-55 MeV

LSND Detector:

-- 1280 phototubes

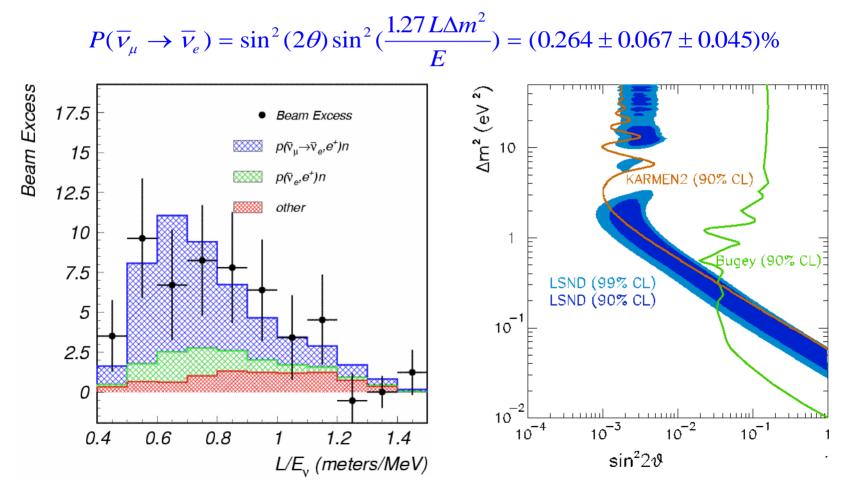
-- 167 tons Liquid Scintillator

Observe an excess of \overline{v}_e : -- 87.9 ± 22.4 ± 6.0 events.

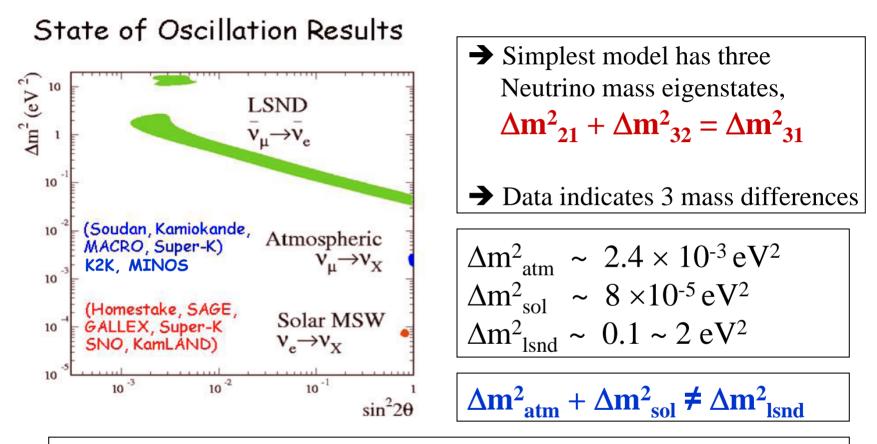
LSND Collab, PRD 64, 112007

The LSND Experiment

→ LSND observed a positive signal(~3.8 σ), but not confirmed.



Physics Motivation



 \rightarrow If the LSND signal does exist, it may imply new physics beyond SM.

LSND Signal: Yes or NO?

The MiniBooNE Experiment

- Proposed in 1998, operating since 2002
- The goal of the MiniBooNE Experiment: to confirm or exclude the LSND result and extend the explored oscillation parameter space

An order of magnitude higher energy (~500 MeV) than LSND (~30 MeV) An order of magnitude longer baseline (~500 m) than LSND (~30 m)

MiniBooNE and LSND have similar L/E, but have different signal, background and systematics.

The MiniBooNE Collaboration

A. A. Aguilar-Arevalo, A. O. Bazarko, S. J. Brice, B. C. Brown,
L. Bugel, J. Cao, L. Coney, J. M. Conrad, D. C. Cox, A. Curioni,
Z. Djurcic, D. A. Finley, B. T. Fleming, R. Ford, F. G. Garcia,
G. T. Garvey, J. A. Green, C. Green, T. L. Hart, E. Hawker,
R. Imlay, R. A. Johnson, P. Kasper, T. Katori, T. Kobilarcik,
I. Kourbanis, S. Koutsoliotas, J. M. Link, Y. Liu, Y. Liu,
W. C. Louis, K. B. M. Mahn, W. Marsh, P. S. Martin, G. McGregor,
W. Metcalf, P. D. Meyers, F. Mills, G. B. Mills, J. Monroe,
C. D. Moore, R. H. Nelson, P. Nienaber, S. Ouedraogo,
R. B. Patterson, D. Perevalov, C. C. Polly, E. Prebys, J. L. Raaf,
H. Ray, B. P. Roe, A. D. Russell, V. Sandberg, R. Schirato,
D. Schmitz, M. H. Shaevitz, F. C. Shoemaker, D. Smith, M. Sorel,
P. Spentzouris, I. Stancu, R. J. Stefanski, M. Sung, H. A. Tanaka,

R. Tayloe, M. Tzanov, M. O. Wascko, R. Van de Water, D. H. White, M. J. Wilking, H. J. Yang, G. P. Zeller, E. D. Zimmerman



2 National Laboratories, 14 Universities, 77 Researchers

University of Alabama Bucknell University University of Cincinnati University of Colorado Columbia University Embry Riddle University Fermi National Accelerator Laboratory Indiana University Los Alamos National Laboratory Louisiana State University University of Michigan Princeton University Saint Mary's University of Minnesota Virginia Polytechnic Institute Western Illinois University Yale University

MiniBooNE Neutrino Beam

Fermilab Proton Booster

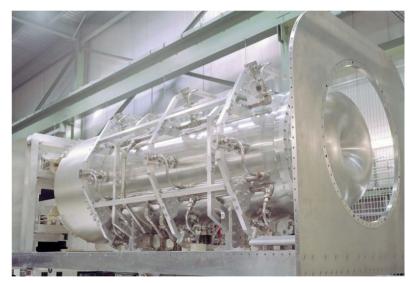


 4×10^{12} protons per 1.6 µs pulse delivered at up to 5 Hz.

Results correspond to $(5.58\pm0.12) \times 10^{20}$ POT

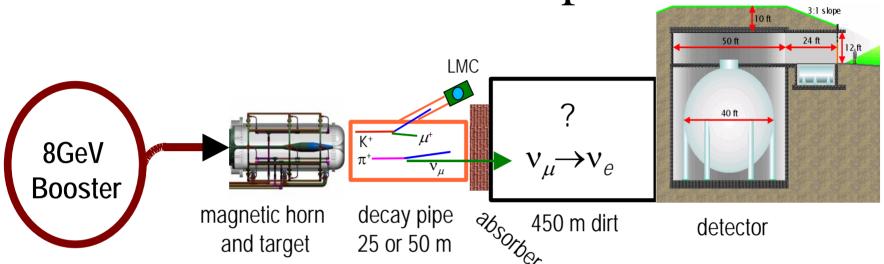
MiniBooNE extracts beam from the 8 GeV Proton Booster

Delivered to a 1.7λ Be target



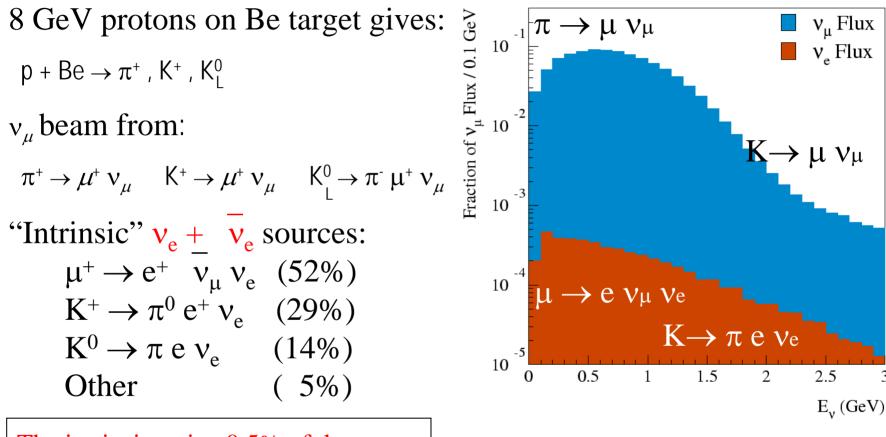
within a magnetic horn (2.5 kV, 174 kA) that (increases the flux by ×6)

The MiniBooNE Experiment



- The FNAL Booster delivers 8 GeV protons to the MiniBooNE beamline.
- The protons hit a 71cm beryllium target producing pions and kaons.
- The magnetic horn focuses the secondary particles towards the detector.
- The mesons decay into neutrinos, and the neutrinos fly to the detector, all other secondary particles are absorbed by absorber and 450 m dirt.
- 5.6E20 POT for neutrino mode since 2002.
- Switch horn polarity to run anti-neutrino mode since January 2006.

MiniBooNE Flux (Geant 4 Simulation)



 $v_{\rm e}/v_{\rm u} = 0.5\%$ Antineutrino content: 6%

The intrinsic v_{ρ} is ~0.5% of the neutrino Flux, it's one of major backgrounds for $v_{\rm u} \rightarrow v_{\rm e}$ search. 3

Modeling Production of Secondary Pions

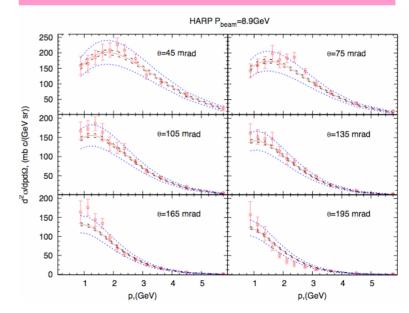
• HARP @ CERN, 8.9 GeV Proton Beam

 $-5\% \lambda$ MB Be target to measure π production

• With E910 @ BNL + previous world data fits

– Basis of current MB π production model





HARP collab., hep-ex/0702024

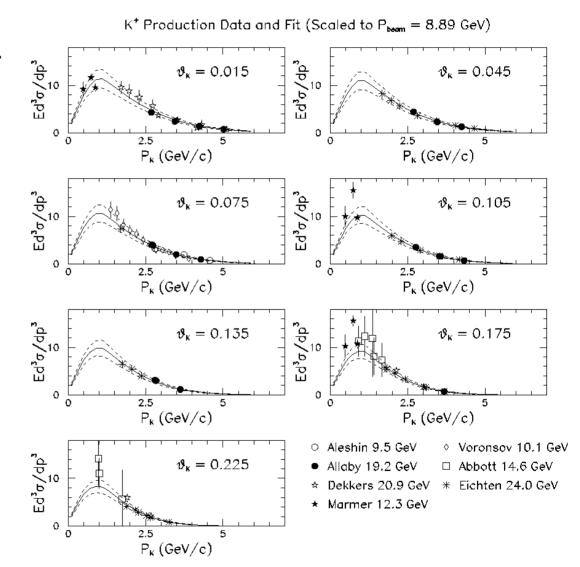
Modeling Production of Secondary Kaons

K⁺ Data from 10 - 24 GeV. Uses a Feynman Scaling Parameterization.

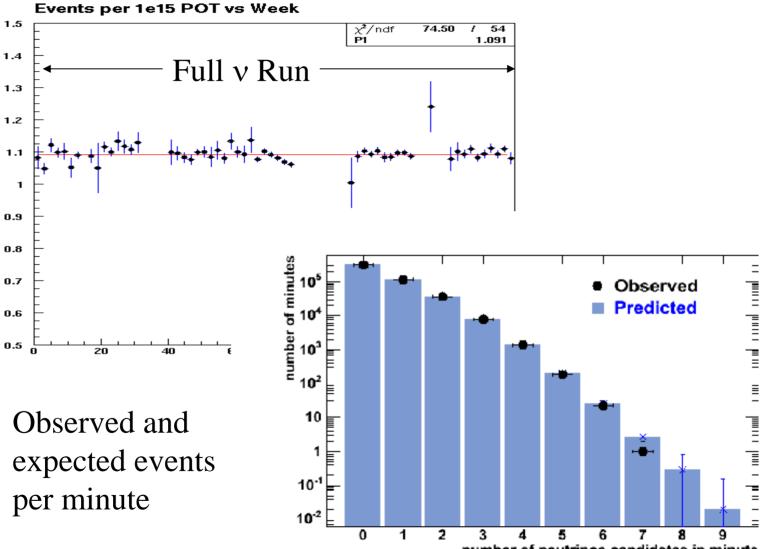
data -- points
dash --total error
(fit ⊕ parameterization)

K⁰ data are also parameterized.

In situ measurement of K⁺ from LMC agrees within errors with parameterization



Stability of Running

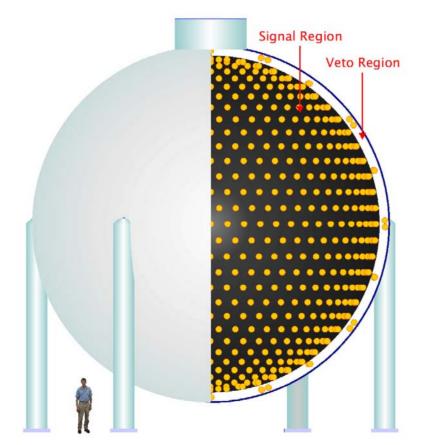


number of neutrinos candidates in minute

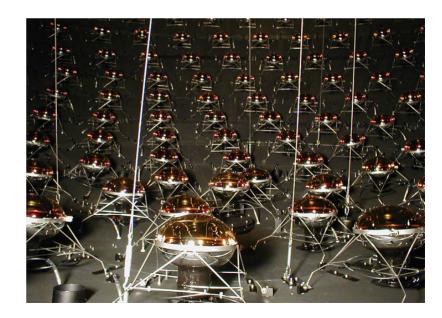
Events in the Detector

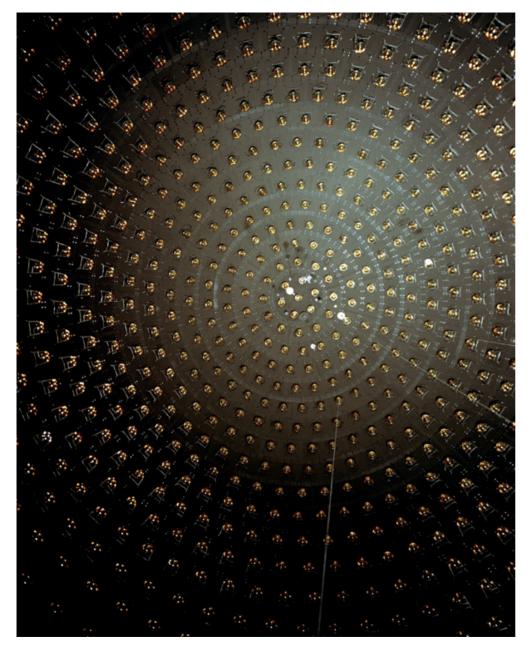
The MiniBooNE Detector

MiniBooNE Detector



- 12m diameter tank
- Filled with 800 tons of ultra pure mineral oil
- Optically isolated inner region with 1280 PMTs
- Outer veto region with 240 PMTs.





10% PMT coverage

Two types of Hamamatsu PMT Tubes: R1408(79%, from LSND) R5912(21%, new)

Charge Resolution: 1.4 PE, 0.5 PE Time Resolution 1.7 ns, 1.1ns



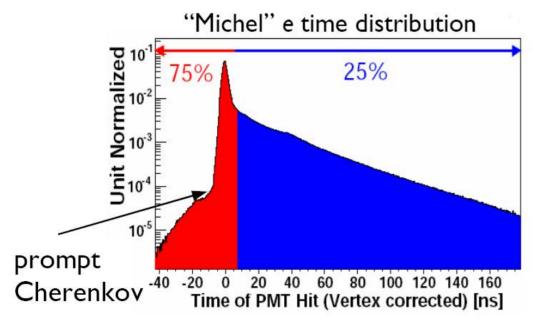
Optical Model

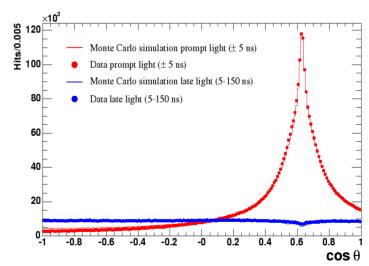
Attenuation length: >20 m @ 400 nm

Detected photons from

- Cherenkov (prompt, directional)
- Scintillation (delayed, isotropic)
- Ratio of prompt/late light ~ 3:1

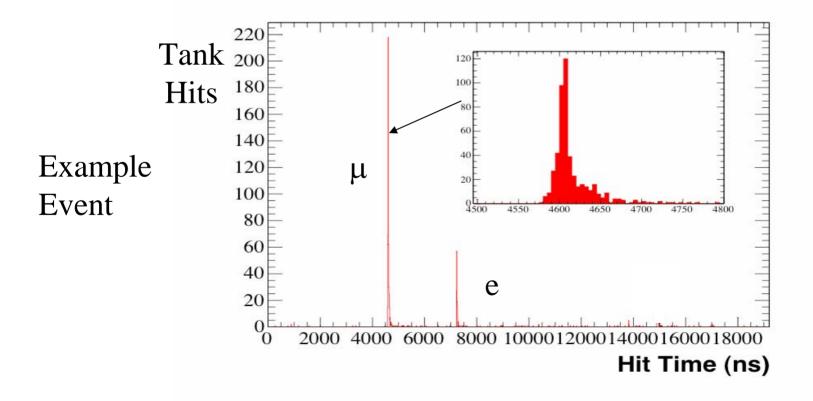
We have developed 39-parameter "Optical Model" based on internal calibration and external measurement



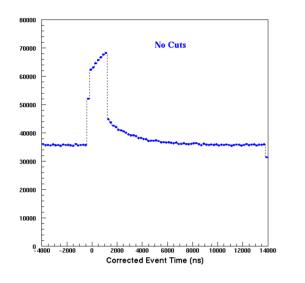


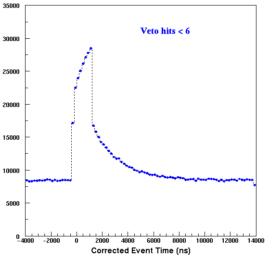
Beam Window

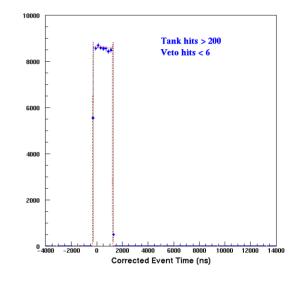
A 19.2 µs beam trigger window encompasses the 1.6 µs spill. Multiple hits within a ~100 ns window form "subevents" Most events are from v_{μ} CC interactions (v_{μ} +n $\rightarrow \mu$ +p) with characteristic two "subevent" structure from stopped $\mu \rightarrow v_{\mu}v_{e}e$



Cuts to Select Neutrino Events





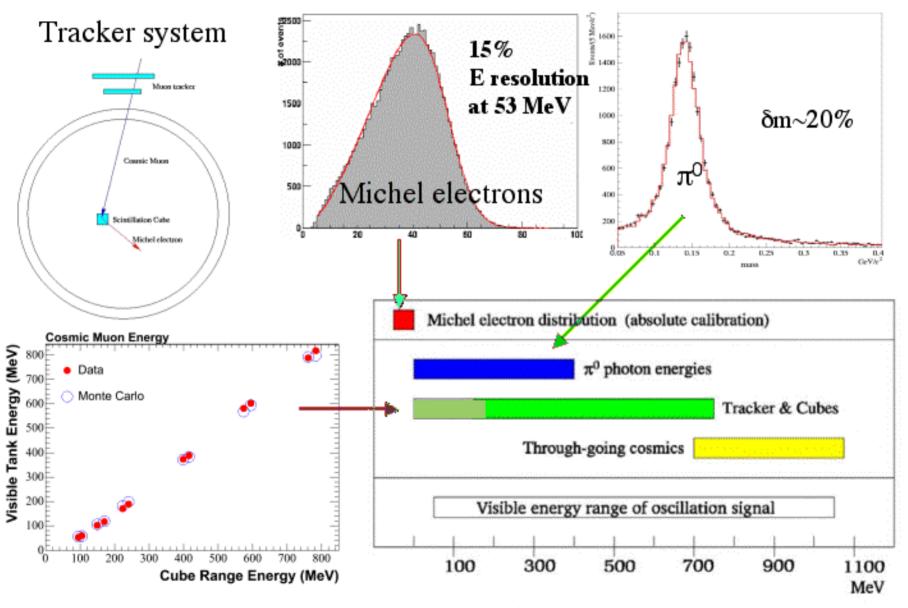


Raw data

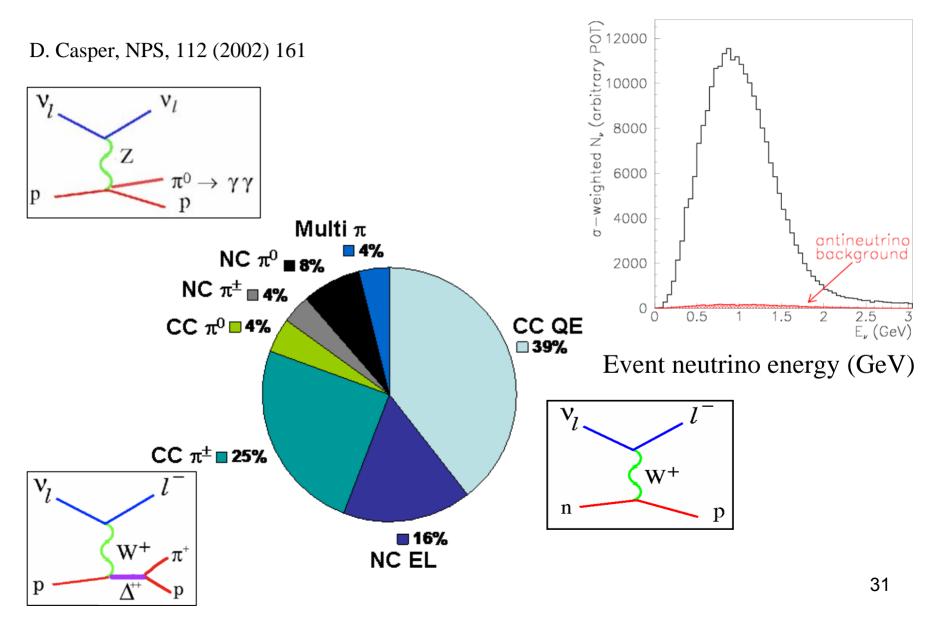
Veto Hits < 6 removes through-going cosmics

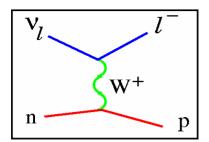
This leaves "Michel electrons" $(\mu \rightarrow \nu_{\mu}\nu_{e}e)$ from cosmics Tank Hits > 200 (equivalent to energy) removes Michel electrons, which have 52.8 MeV endpoint

Calibration Sources



Nuance MC Event Rates





CCQE Events $\theta_{\mu or e}$

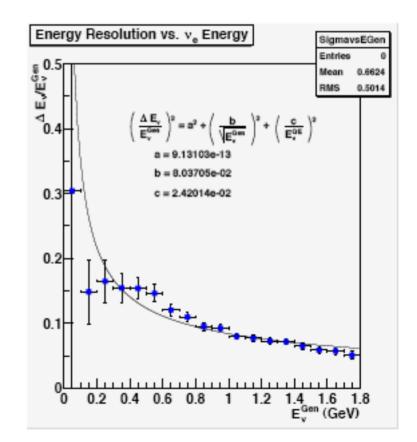
n

CCQE (Charged Current Quasi-Elastic) 39% of total

- Events are "clean" (few particles)
- Energy of the neutrino can be reconstructed

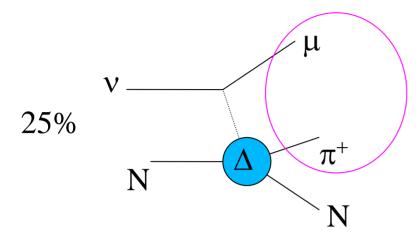
$$E_{\mathcal{V}}^{QE} = \frac{1}{2} \frac{2M_p E_\ell - m_\ell^2}{M_p - E_\ell + \sqrt{(E_\ell^2 - m_\ell^2)} \cos\theta_\ell}$$

Reconstructed from: Scattering angle Visible energy (E_{visible})



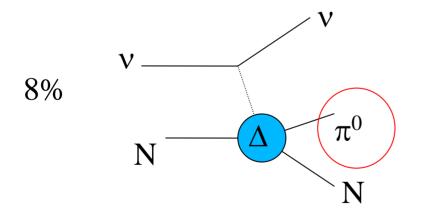
μ or e

Events Producing Pions



 $CC\pi^+$

Easy to tag due to 3 subevents. Not a substantial background to the oscillation analysis.



(also decays to a single photon with 0.56% probability)

$NC\pi^0$

The π^0 decays to 2 photons, which can look "electron-like" mimicking the signal...

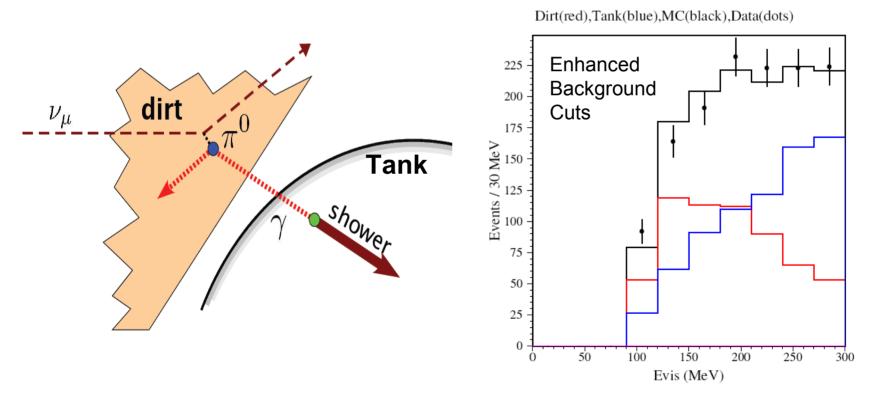
<1% of π^0 contribute to background.

External Sources of Background

Cosmic Rays: measured from out-of-beam data: 2.1 ± 0.5 events

"Dirt" Events:

v interactions outside of the detector $N_{data}/N_{MC} = 0.99 \pm 0.15$



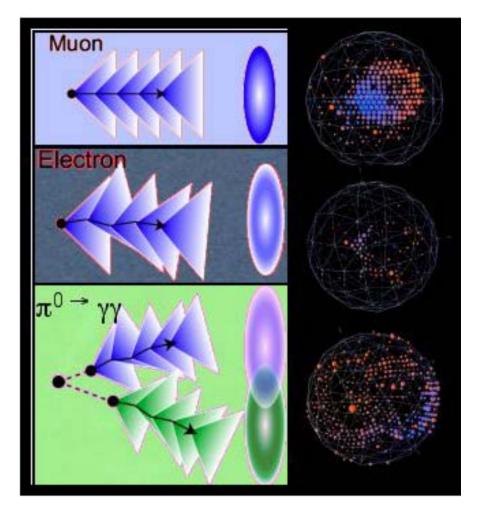
MiniBooNE Event Types

Muons: Produced in most CC events. Usually 2 subevent.

Electrons: Tag for $v_{\mu} \rightarrow v_{e}$ CCQE signal. 1 subevent, fuzzy ring.

π^0 s:

Can form a background if one photon is weak or exits tank. In NC π^0 case, 1 subevent.



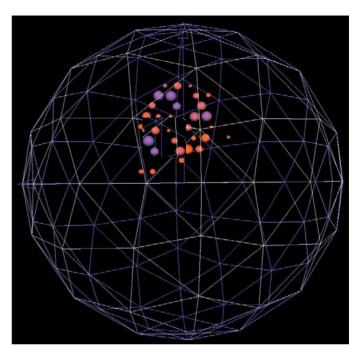
Two Independent Analyses

Blindness Analysis

MiniBooNE is searching for a small but distinctive event signature (electron)

In order to maintain blindness,

Electron-like events were sequestered,



Leaving ~99% of the in-beam events available for study.

Rule for cuts to sequester events: $<1\sigma$ signal outside of the box

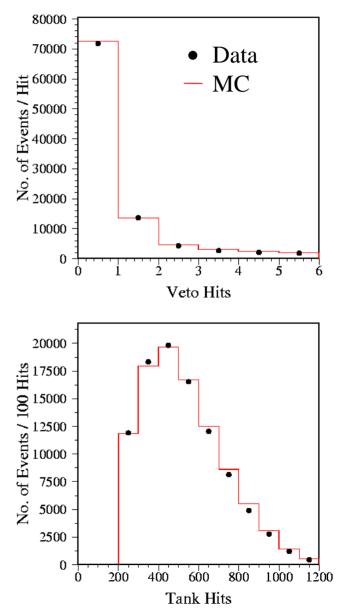
Low level information which did not allow particle-id was available for all events.

Pre-selection Cuts

Both Algorithms and all analyses presented here share "hit-level pre-cuts":

Only 1 subevent Number of Veto hits < 6 Number of Tank hits > 200

And a track radius precut: R < 500 cm



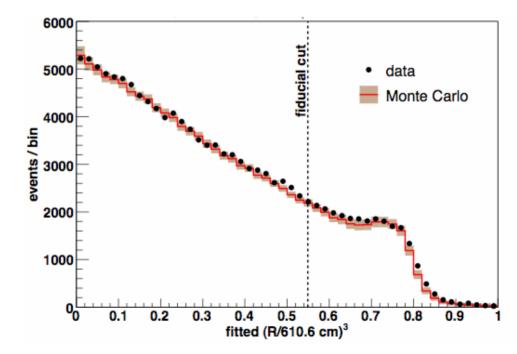
Track-Based (TB) Analysis

Philosophy:

→Uses detailed, direct reconstruction of particle tracks, and ratio of fit likelihoods to identify particles.

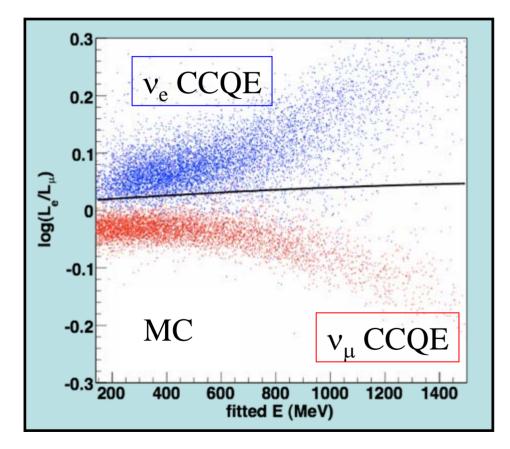
Each event is characterized by 7 reconstructed variables: vertex (*x*,*y*,*z*), time, energy, and direction (θ , ϕ) \Leftrightarrow (U_x , U_y , U_z).

Resolutions: vertex: 22 cm direction: 2.8° energy: 11%



e / μ Separation

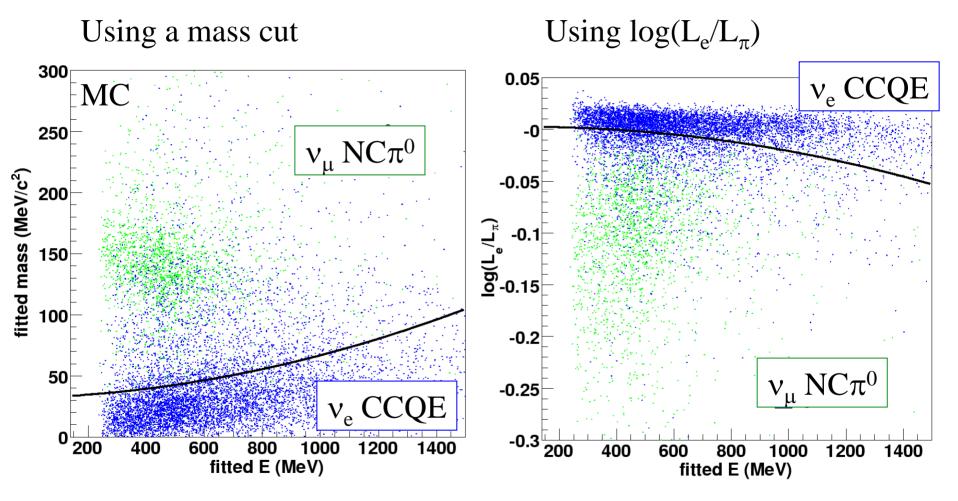
 $log(L_e/L_{\mu})>0$ favors electron-like hypothesis



Separation is clean at high energies where muon-like events have long tracks.

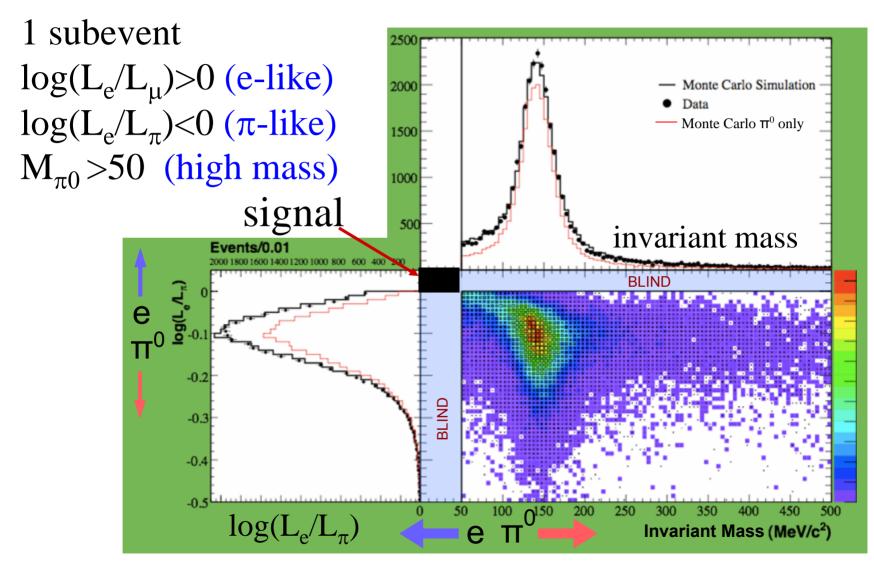
Analysis cut was chosen to maximize the $\nu_{\mu} \rightarrow \nu_{e}$ sensitivity

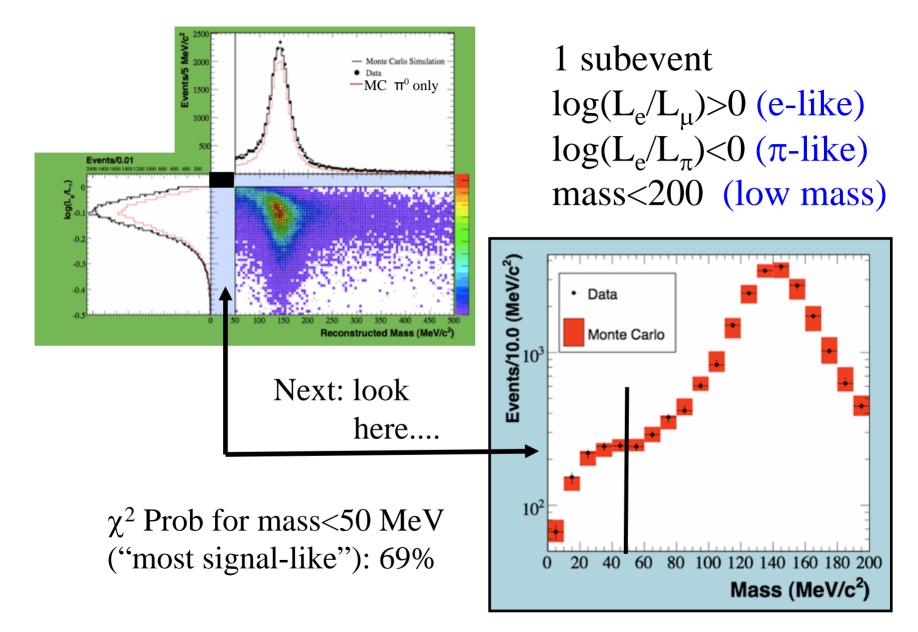
e / π^0 Separation



Cuts were chosen to maximize $v_{\mu} \rightarrow v_{e}$ sensitivity

Testing e / π^0 Separation using data

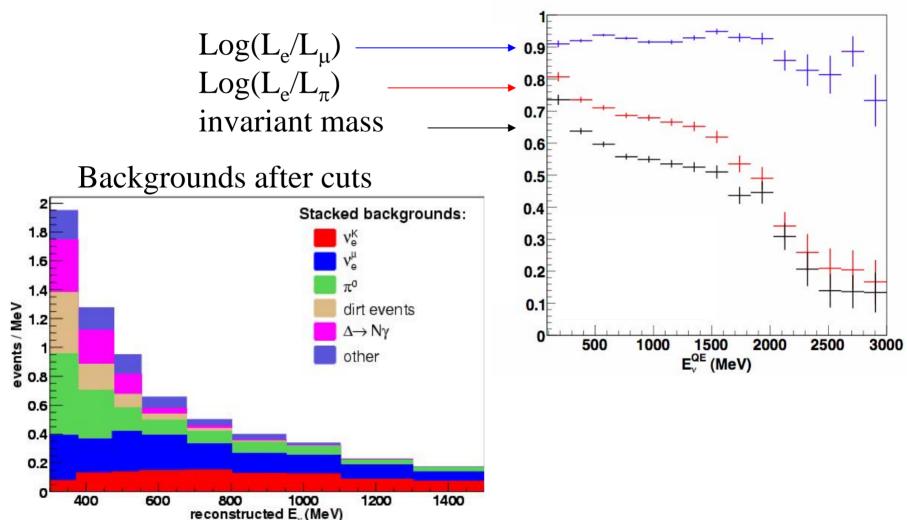




Summary of Track-Based Cuts

"Precuts" +

Efficiency:

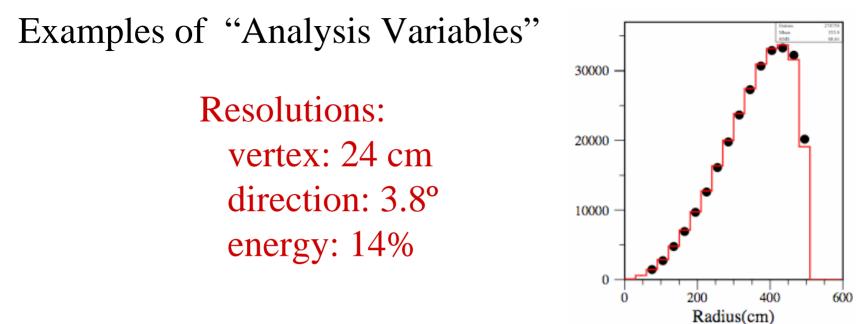


Boosted Decision Trees (BDT) Analysis

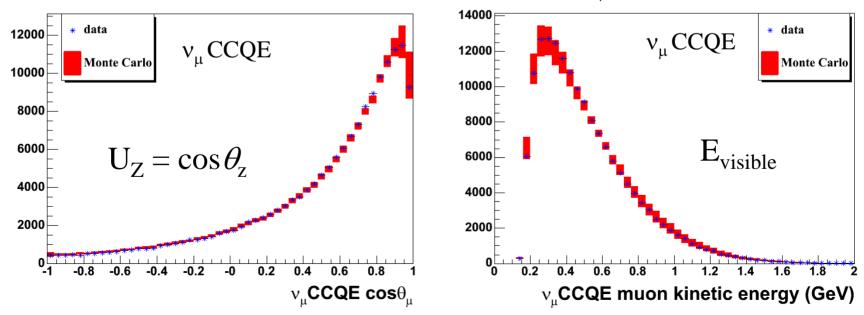
Philosophy:

→Construct a set of low-level analysis variables which are used to make a series of cuts to classify the events – decision tree.

➔Boosted Decision Trees combine many trees (weak classifiers) to build a powerful committee to improve signal efficiency.

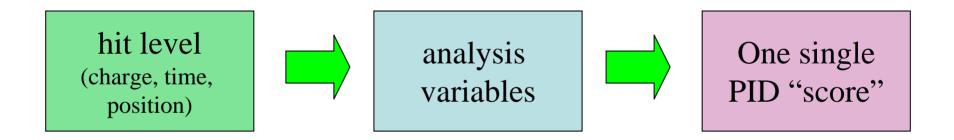


Reconstructed quantities which are inputs to E_v^{QE}

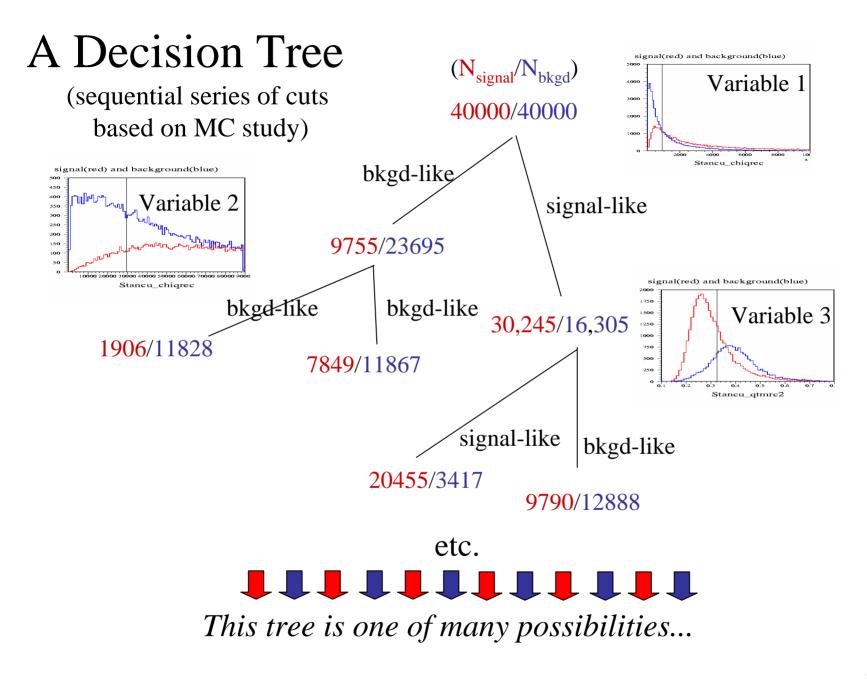


Many Variables → A Single PID Variable Boosted Decision Trees

"A procedure that combines many weak classifiers to form a powerful committee"



Byron P. Roe, Hai-Jun Yang, Ji Zhu *et al.*, NIM A543 (2005) 577, physics/0408124 Hai-Jun Yang, Byron P. Roe, Ji Zhu, NIM A555 (2005) 370, physics/0508045



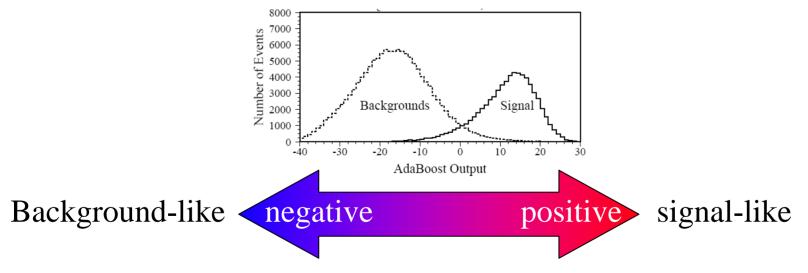
A set of decision trees can be developed, each re-weighting the events to enhance identification of backgrounds misidentified by earlier trees ("boosting")

For each tree, the data event is assigned

+1 if it is identified as signal,

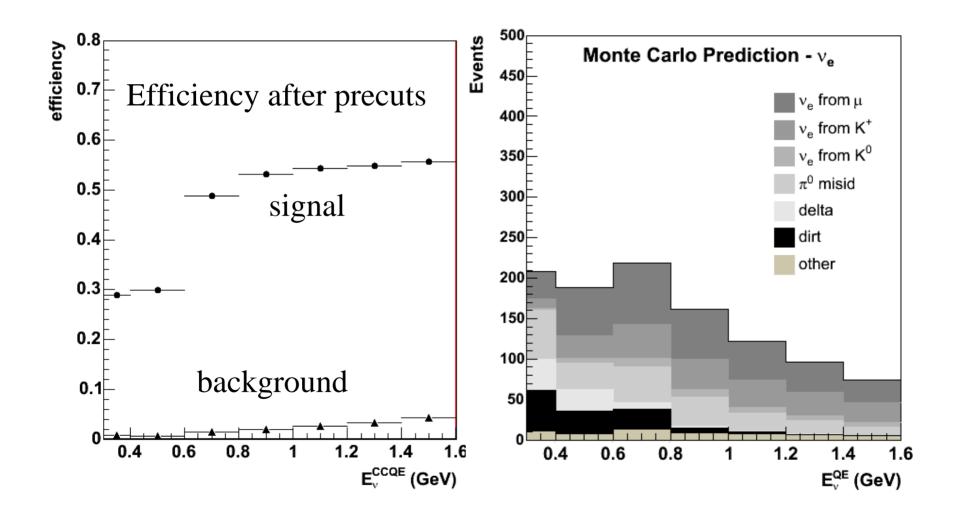
-1 if it is identified as background.

The total for all trees is combined into a "score"



BDT Efficiency and backgrounds after cuts:

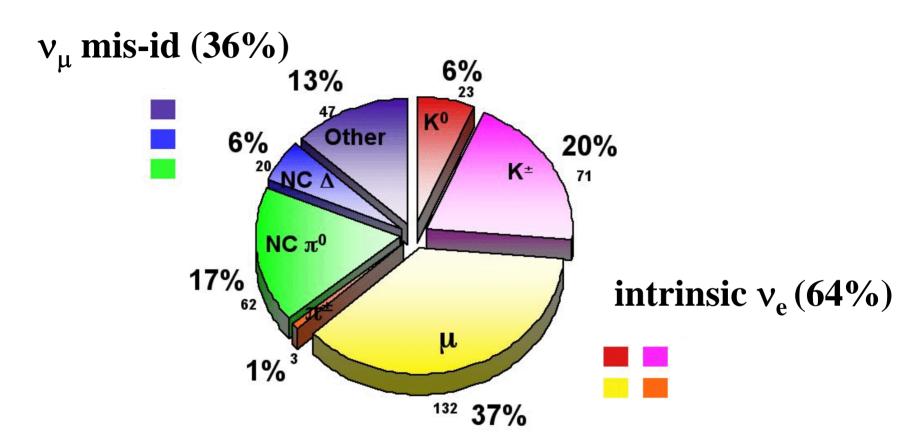
Analysis cuts on PID score as a function of Energy



Sources of Uncertainty

Source of	Track Based /Boosted	Checked or Constrained 1	
Uncertainty	Decision Trees	by MB data	tying
On v_e background	error in %		v_{e} to v_{μ}
Flux from π^+/μ^+ decay	6.2 / 4.3	\checkmark	
Flux from K ⁺ decay	3.3 / 1.0	\checkmark	
Flux from K ⁰ decay	1.5 / 0.4	\checkmark	
Target and beam models	2.8 / 1.3	\checkmark	
v-cross section	12.3 / 10.5	\checkmark	
NC π^0 yield	1.8 / 1.5	\checkmark	
External interactions ("Dirt")	0.8 / 3.4	\checkmark	
Optical model	6 .1 / 10.5	\checkmark	
DAQ electronics model	7.5 / 10.8	\checkmark	

Main Backgrounds



Predictions of the backgrounds are among the nine sources of significant error in the analysis

Summary of predicted backgrounds for the final MiniBooNE result

Process	Number of Events
ν_{μ} CCQE	10
$ u_{\mu}e ightarrow u_{\mu}e$	7
Miscellaneous ν_{μ} Events	13
NC π^0	62
$NC \Delta \rightarrow N\gamma$	20
NC Coherent & Radiative γ	< 1
Dirt Events	17
ν_e from μ Decay	132
ν_e from K^+ Decay	71
ν_e from K_L^0 Decay	23
ν_e from π Decay	3
Total Background	358
$0.26\% \ \nu_{\mu} \rightarrow \nu_{e}$	(example signal) ¹⁶³

The Neutrino Oscillation Results

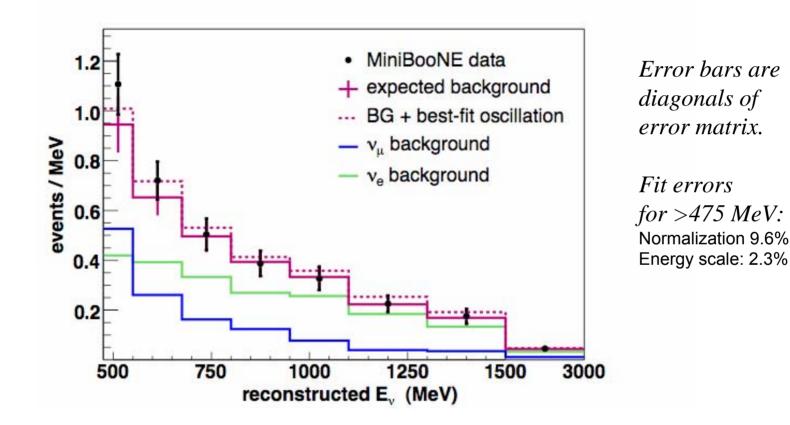
Track-based Analysis Results

Counting Experiment: $475 < E_v^{QE} < 1250 \text{ MeV}$

data: 380 events expectation: 358 ± 19 (stat) ± 35 (sys) events

> significance: 0.55 σ

Track Based energy dependent fit results: Data are in good agreement with background prediction.



Best Fit (dashed): $(\sin^2 2\theta, \Delta m^2) = (0.001, 4 \text{ eV}^2)$

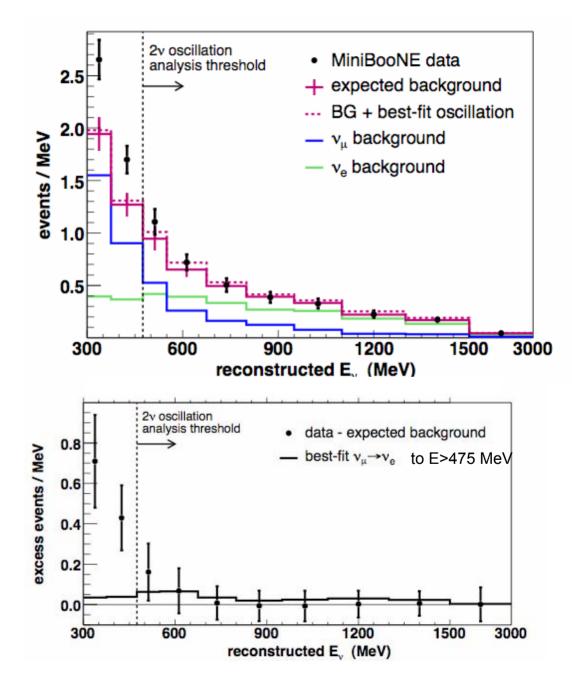
Low E Excess

Report the full range: $300 < E_v^{QE} < 3000 \text{ MeV}$

96 ± 17 ± 20 events above background, for $300 < E_v^{QE} < 475 MeV$

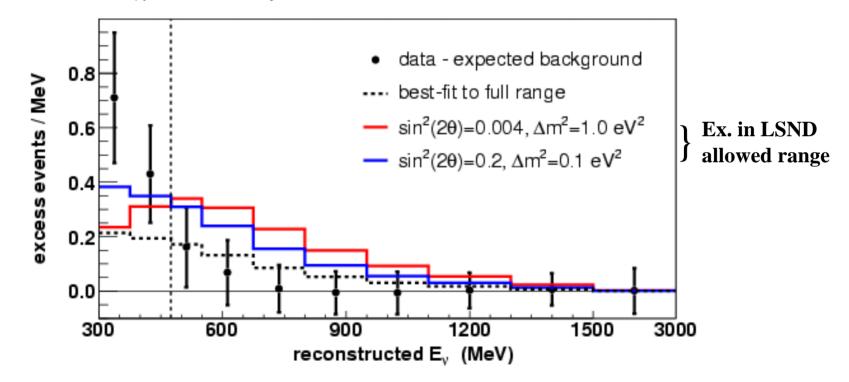
Deviation: 3.7σ

Background-subtracted:



Fit to the > 300 MeV range

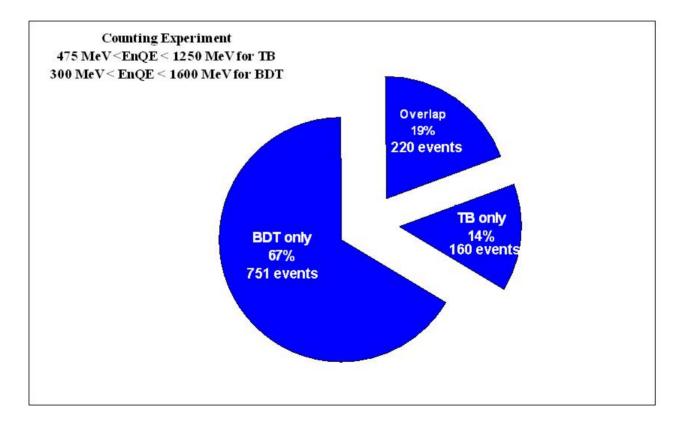
Best Fit (dashed): $(\sin^2 2\theta, \Delta m^2) = (1.0, 0.03 \text{ eV}^2)$ χ^2 Probability: 18%



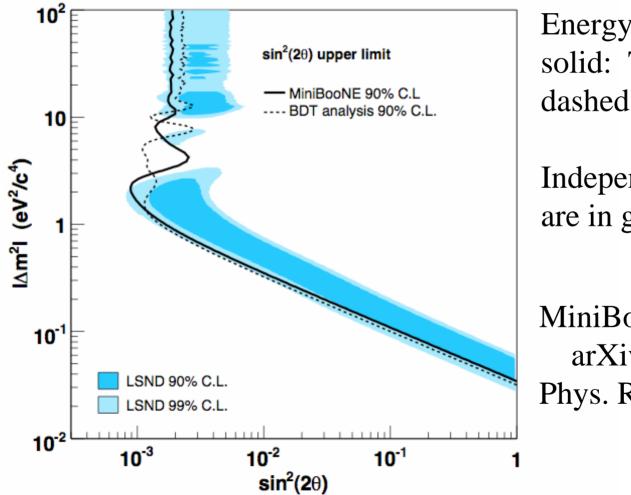
 \rightarrow Low E excess cannot be explained with $v_{\mu} \rightarrow v_{e}$ oscillation.

Boosted Decision Trees Analysis

Counting Experiment: $300 < E_v^{QE} < 1600 \text{ MeV}$ data: 971 events expectation: $1070 \pm 33 \text{ (stat)} \pm 225 \text{ (sys)}$ events significance: -0.38σ



MiniBooNE First Results show no evidence for $v_{\mu} \rightarrow v_{e}$ appearance-only oscillations

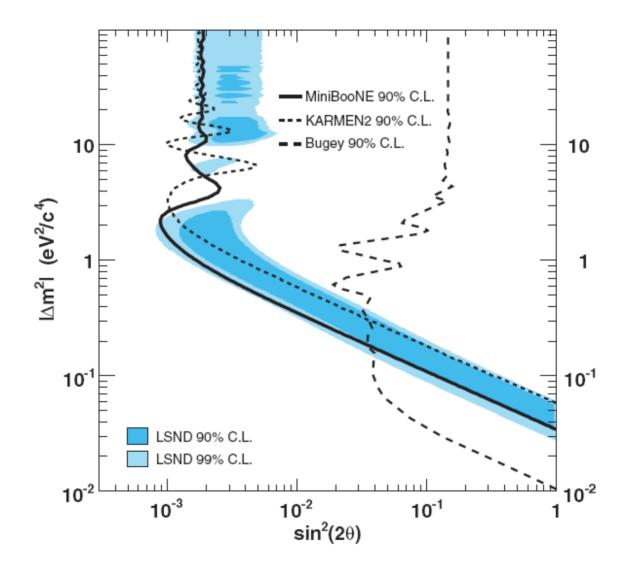


Energy-fit analysis: solid: TB dashed: BDT

Independent analyses are in good agreement.

MiniBooNE first results arXiv:0704.1500 Phys. Rev. Lett. 98, 231801

Exclusion Limits (90% CL)



LSND signal: Yes or NO ? \rightarrow NO

"As in many particle physics experiments, we have a result that answers some questions and raises others," said MiniBooNE co-spokesperson William Louis.

"It clears one mystery but it leaves us with a puzzle that is important to understand," said Fermilab Director Pier Oddone.

MiniBooNE low E excess: Yes or NO?

Backup Slides

Future Plans:

Many more papers supporting this analysis will follow, *in the very near future:*

 v_{μ} CCQE production (arXiv:0706.0926) π^{0} production MiniBooNE-LSND-Karmen joint analysis

We are pursuing further analyses of the neutrino data, including...

an analysis which combines TB and BDT, more exotic models for the LSND effect.

We are working hard to understand the low E excess.

MiniBooNE is presently taking data in antineutrino mode.

Low E Excess (current status)

- ? Instrumental background: NO
- ? Study the excess with both Track and Boosting analysis
- Are they consistent in energy and numbers: YES
- Are there any reco issues (sidebands, etc): TB NO
- Excess down to 200 MeV with systematic errors.
- Is the excess electron/gamma-ray like: YES
- ? Is it a source of background
- Dirt/Delta rays: NO
- Pion or muon mis-id (including brem): NO
- Evis and UZ (low stats) shapes favor numu background(?)
- Photonuclear: ~20% of excess. Needs systematic errors.
- Other source of backgrounds? Still Investigating
- ? Do other data sets have low E excess
- NuMI -- different source, energy, baseline, backgrounds:
- appears consistent in energy and scales with nue rate.

Low E Excess

- Future Work
- ? Continue checking reconstruction/PID.
- ? Study PN contribution to excess, confirm in data.
- ? Study new sources of backgrounds.
- ? Understand systematics below ~200 MeV
- Apply to other distributions.
- ? Continue investigating forward excess.
- ? NuMI analysis matured, study correlations with
- MB excess, i.e. excess/numu, excess/nue,
- excess/pi0, etc. Does it scale with anything?
- ? Check excess with CCPi+ sample
- ? Check excess in Horn-off and anti-neutrino data.
- Look for POT or (flux*xsec) scaling.

One, Two or Three Sterile Neutrinos?

- Michael Maltoni, arXiv:0711.2018
- Parameter goodness of fit (PG) test to appearance and disappearance datasets from MiniBooNE, LSND, KARMEN and NOMAD experiments.

PG = 4.0×10^{-6} for (3+1 sterile v) model PG = 4.8×10^{-5} for (3+2 sterile v) model

→ Severe tension between different datasets. With present experimental results, (3+1), (3+2) and (3+3) neutrino oscillation schemes is NOT possible to explain the LSND signal in terms of sterile neutrinos.

A MB-LSND-KARMEN-Bugey Compatibility Test

$$\chi_0^2 = \frac{(z_{MB} - z_0)^2}{\sigma_{MB}^2} + \frac{(z_{LSND} - z_0)^2}{\sigma_{LSND}^2}$$

• For each Δm^2 , determine the MB and LSND measurement:

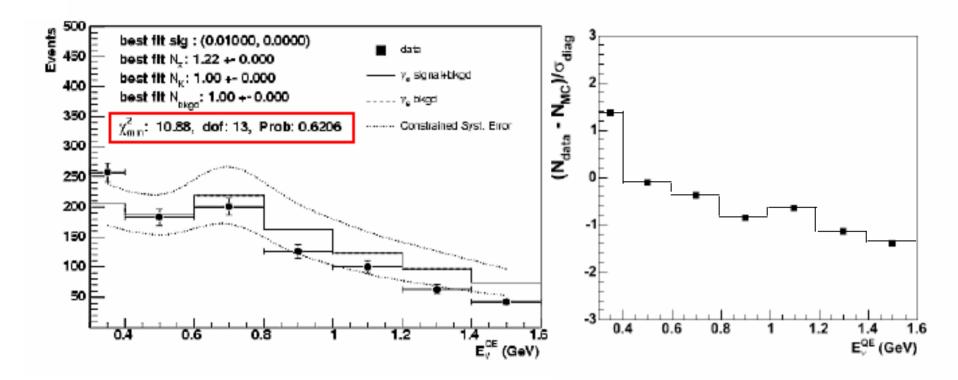
$$z_{MB} \pm \delta z_{MB}$$
, $z_{LSND} \pm \delta z_{LSND}$, $z_{K} \pm \delta z_{K}$, $z_{B} \pm \delta z_{B}$
where $z = \sin^{2}(2\theta)$ and δz is the 1σ error

- For each Δm^2 , form χ^2 between MB and LSND measurement
- Find z_0 that minimizes χ^2

(weighted average of two measurements) and this gives χ^2_{min}

- Find probability of χ^2_{min} for 1 dof; this is the joint compatibility probability for this Δm^2
- \rightarrow The combined compatible is at 8.6% C.L.

Boosted Decision Tree E_v^{QE} data/MC comparison:



Error Matrix Elements:

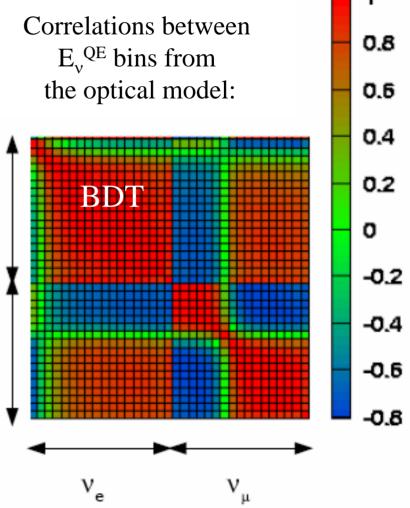
$$E_{ij} \approx \frac{1}{M} \sum_{\alpha=1}^{M} \left(N_i^{\alpha} - N_i^{MC} \right) \left(N_j^{\alpha} - N_j^{MC} \right)$$

 ν_{μ}

- N is number of events passing cutsMC is standard monte carlo
- α represents a given multisim
- M is the total number of multisims
- i,j are E_v^{QE} bins

Total error matrix is sum from each source.

TB: v_e -only total error matrix BDT: v_{μ} - v_e total error matrix



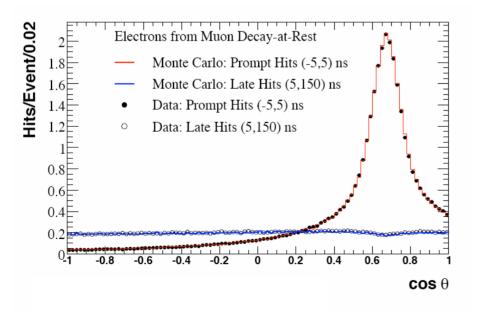
Example: Cross Section Uncertainties (Many are common to v_{μ} and v_{e} and cancel in the fit)

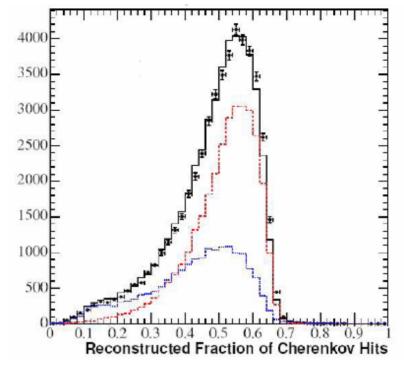
M_A^{QE} , e_{lo}^{sf} QE σ norm QE σ shape ν_e/ν_μ QE σ	6%, 2% (stat + bkg only) 10% function of E_v function of E_v	determined from MiniBooNE v _µ QE data
NC π^0 rate M _A ^{coh} , coh σ $\Delta \rightarrow N\gamma$ rate	function of π^0 mom $\pm 25\%$ function of γ mom + 7% BF	determined from MiniBooNE $v_{\mu} NC \pi^{0} data$
E _B , p _F	9 MeV, 30 MeV	
Δs	10%	determined
$M_A^{1\pi}$	25%	from other
${ m M}_{ m A}^{ m N\pi}$	40%	experiments
DIS σ	25%	

Example: Optical Model Uncertainties

39 parameters must be varied

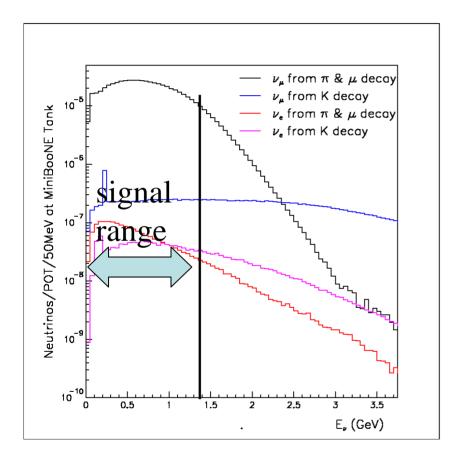
Allowed variations are set by the Michel calibration sample





To understand allowed variations, we ran 70 hit-level simulations, with differing parameters. \Rightarrow "Multisims"

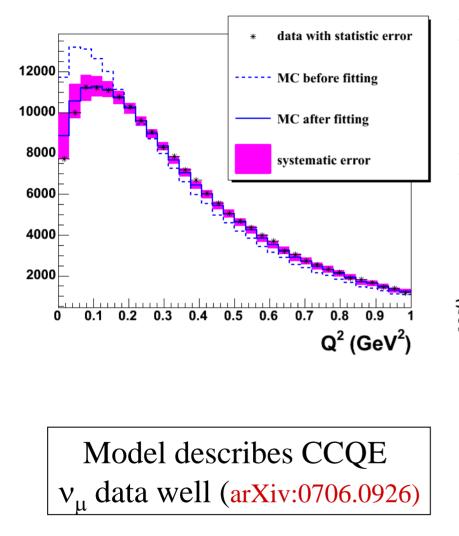
K⁺ and K⁰ Decay Backgrounds



At high energies > 1.5 GeV, above "signal range" v_{μ} and " v_{e} -like" events are largely due to kaon decay

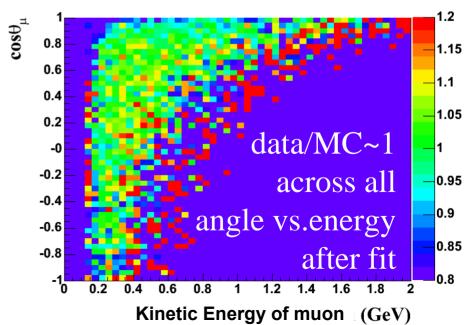
By measuring high energy box events (>1.5 GeV) to estimate K^+ & K^0 production rate.

Nuance Parameters (v_{μ} CCQE)

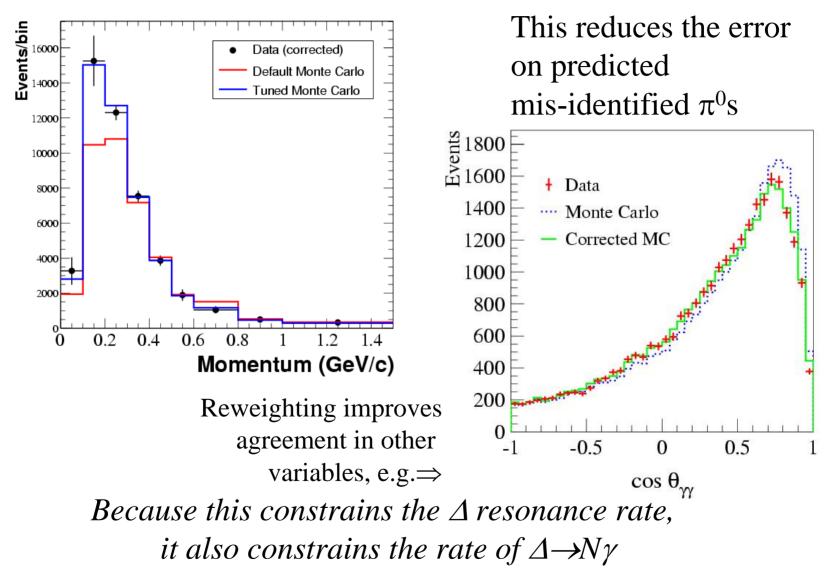


From Q² fits to MB ν_{μ} CCQE data: M_A^{eff} -- effective axial mass E_{lo}^{SF} -- Pauli Blocking parameter

From electron scattering data: E_b -- binding energy p_f -- Fermi momentum



We constrain π^0 production using data from our detector



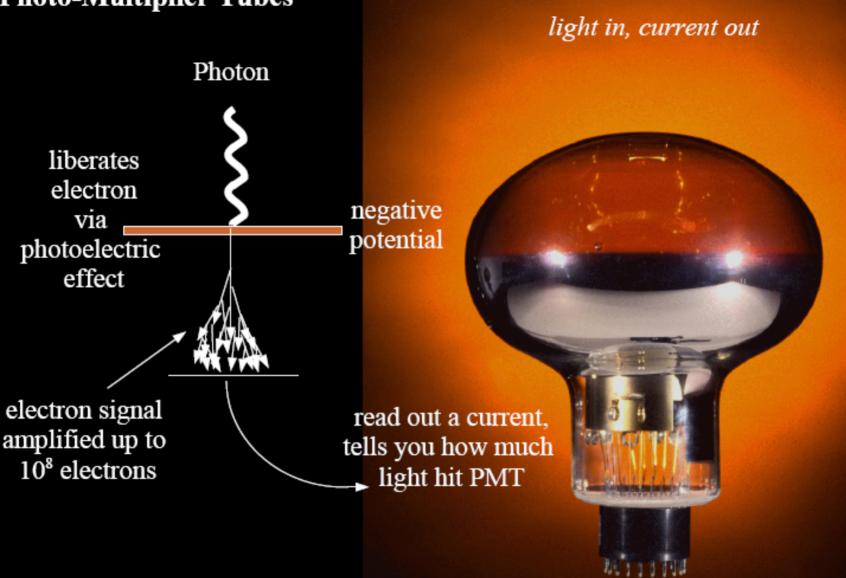
Other Single Photon Sources

Neutral Current: $v + N \rightarrow v + N + \gamma$ negligibleFrom Efrosinin, hep-ph/0609169,
calculation checked by Goldman, LANLnegligible

Charged Current < 6 events @ 95% CL $\nu + N \rightarrow \mu + N' + \gamma$ where the presence of the γ leads to mis-identification

Use events where the μ is tagged by the michel e^{-,} study misidentification using BDT algorithm.

Photo-Multiplier Tubes



number of photons = $10^{-8} x$ number of electrons out

Neutrino Oscillations

Maki-Nakagawa-Sakata matrix (MNS matrix – neutrino mixing matrix)

$$|\nu_{\alpha}\rangle = \sum_{i} U_{\alpha i}^{*} |\nu_{i}\rangle$$
 $|\nu_{\alpha}\rangle$ is a neutrino with definite flavor. $\alpha = e, \mu, \tau$.
 $|\nu_{i}\rangle = \sum_{\alpha} U_{\alpha i} |\nu_{\alpha}\rangle$ $|\nu_{i}\rangle$ is a neutrino with definite mass. $i = 1, 2, 3$.

phase factor δ is non-zero if neutrino oscillation violates CP symmetry

phase factors α_1 and α_2 are non-zero if neutrinos are Majorana particles

$$\begin{split} U &= \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix} \begin{bmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{bmatrix}, \end{split}$$